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The History of the Cavalieri Estimator – Post, Present, and Future

The Cavalieri estimator is one of the most fundamental workhorses of the stereologist. The estimator is used to estimate the volume of a region of interest—such as an organ or one or more of its subcompartments (e.g. the grey matter of the brain). As commonly known, in its basic form, the Cavalieri estimator requires that the whole region of interest is available for sampling and that the region of interest is well-defined with identifiable boundaries. The region is cut in its entirety, with a uniformly random position of the set of cuts, into equidistant parallel slices of constant thickness T . The volume V of the region of interest is then estimated as:

$$V := T \cdot \sum A_i$$

where the sum is across the individual areas A_i of the region of interest as observed at the cut surfaces of the individual slices. The areas may be assessed in various ways, e.g. estimated by point counting. The estimator is unbiased. I.e., by repetition, the mean of the re-estimates converges on the true mean.

The basic geometric concepts behind the estimator have been known for millennia – they were familiar to Archimedes of Syracuse (c. 287–212 BC), the Chinese mathematicians Liu Hui (c. 236) and Zu Geng (480–525) as well as to Bonaventura Cavalieri (1598–1647). The stochastic part of the estimator and its finer details evolved among stereologists during the 20th century. The modern estimator was named *The Cavalieri Estimator* in honor of Cavalieri and his famous theorem known as *Cavalieri's principle*.

In the recent years, the development of methods to predict the precision of the estimator has been a hot topic in stereological research. Also, various ways to soften the requirements of the estimator has been investigated. Thus, it has been shown that (under some constraints) the estimator is still unbiased even in the case of non-equidistant, uniformly random parallel cuts.

In my presentation, I will review the past and present of the Cavalieri Estimator and point to some areas of future development.