Abstract

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Optimal non-signalling violations via tensor norms

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During this talk we will characterize the set of bipartite non-signalling probability distributions in terms of tensor norms. Using this characterization we will give optimal upper and lower bounds on Bell inequality violations when non-signalling distributions are considered.

More specifically, in the Bell scenario, the tensor $(P(a,b|x,y))_{xyab}$ denotes the probability distribution of two spacially separated parties (Alice and Bob) who perform two different measurements x and y to obtain outputs a and b, respectively. The use of classical resources makes this tensor to lie in $\mathcal{L}$, the set of classical probability distributions; while if they share a quantum state and perform quantum measurements on it, the tensor lies in $\mathcal{Q}$, the set of quantum probability distributions; finally, we can include both of them in a more general set, $\mathcal{NS}$, known as the non-signalling set, in which marginal probability distributions $Q_1(a|x) = \sum_b P(a,b|x,y)$ and $Q_2(b|y) = \sum_a P(a,b|x,y)$ are well defined.

A natural quantification of how different the sets of $\mathcal{L}$, $\mathcal{Q}$ and $\mathcal{NS}$ are can be done by means of the so called Bell inequality violations. More precisely, if $\mathcal{A},\mathcal{B} \in \{\mathcal{L},\mathcal{Q},\mathcal{NS}\}$ and $M \in \mathbb{R}^{N^2K^2}$ is any tensor, let us denote

$$\omega_{\mathcal{A}}(M) = \sup_{P\in\mathcal{A}} |\langle M,P\rangle|,$$

where the dual action is given by $\langle M,P\rangle = \sum_{x,y;a,b=1}^{N,K} M_{x,y}^{a,b} P(a,b|x,y)$. Then, there is an $\mathcal{B}-\mathcal{A}$ *Bell violation* for M if $\omega_{\mathcal{A}}(M)/\omega_{\mathcal{B}}(M) > 1$.

Although during the talk some other results will be shown, the main theorems are:

Theorem 1: Given a general tensor $M \in \mathbb{R}^{N^2K^2}$. Then,

$$\frac{\omega_{\mathcal{NS}}(M)}{\omega_{\mathcal{L}}(M)} \leq O(\min\{N, \sqrt{NK}\}).$$

Theorem 2: For every natural number n there exists a pointwise non-negative tensor $G_n \in \mathbb{R}^{N^2K^2}$, $N{=}K{=}n$, such that

$$\frac{\omega_{\mathcal{NS}}(G)}{\omega_{\mathcal{L}}(G)} \geq D\frac{n}{\log n},$$

where D is a universal constant.

The aim of studying this ratio, the classical-non-signalling Bell violation, should be seen as a way to study the ultimate limitations of any *meaningful* physical theory. During the talk we will compare these upper and lower bounds with the existing ones for the quantum-classical Bell violation. Then, a remarkable results follows: although we can not say yet that the largest quantum Bell violation is comparable to the largest non-signalling Bell violation, our bounds show that this result is indeed very plausible. This emphasizes the idea that, in some sense, the theory of quantum mechanic is as non-local as any other physical theory can be.