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**Optimal non-signalling violations via tensor norms**

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During this talk we will characterize the set of bipartite non-signalling probability distributions in terms of tensor norms. Using this characterization we will give optimal upper and lower bounds on Bell inequality violations when non-signalling distributions are considered.

More specifically, in the Bell scenario, the tensor \((P(a,b|x,y))_{xyab}\) denotes the probability distribution of two spatially separated parties (Alice and Bob) who perform two different measurements \(x\) and \(y\) to obtain outputs \(a\) and \(b\), respectively. The use of classical resources makes this tensor to lie in \(L\), the set of classical probability distributions; while if they share a quantum state and perform quantum measurements on it, the tensor lies in \(Q\), the set of quantum probability distributions; finally, we can include both of them in a more general set, \(NS\), known as the non-signalling set, in which marginal probability distributions \(Q_1(a|x) = \sum_b P(a,b|x,y)\) and \(Q_2(b|y) = \sum_a P(a,b|x,y)\) are well defined.

A natural quantification of how different the sets of \(L\), \(Q\) and \(NS\) are can be done by means of the so called Bell inequality violations. More precisely, if \(A,B \in \{L,Q,NS\}\) and \(M \in \mathbb{R}^{N^2K^2}\) is any tensor, let us denote

\[
\omega_A(M) = \sup_{P \in A} |\langle M, P \rangle|,
\]

where the dual action is given by \(\langle M, P \rangle = \sum_{x,y,a,b=1}^{N,K} M_{x,y}^{a,b} P(a,b|x,y)\). Then, there is an \(B - A\) Bell violation for \(M\) if \(\omega_A(M)/\omega_B(M) > 1\).

Although during the talk some other results will be shown, the main theorems are:
Theorem 1: Given a general tensor $M \in \mathbb{R}^{N^2 K^2}$. Then,
\[
\frac{\omega_{NS}(M)}{\omega_L(M)} \leq O(\min\{N, \sqrt{NK}\}).
\]

Theorem 2: For every natural number $n$ there exists a pointwise non-negative tensor $G_n \in \mathbb{R}^{N^2 K^2}$, $N=K=n$, such that
\[
\frac{\omega_{NS}(G_n)}{\omega_L(G_n)} \geq D \frac{n}{\log n},
\]
where $D$ is a universal constant.

The aim of studying this ratio, the classical-non-signalling Bell violation, should be seen as a way to study the ultimate limitations of any meaningful physical theory. During the talk we will compare these upper and lower bounds with the existing ones for the quantum-classical Bell violation. Then, a remarkable results follows: although we can not say yet that the largest quantum Bell violation is comparable to the largest non-signalling Bell violation, our bounds show that this result is indeed very plausible. This emphasizes the idea that, in some sense, the theory of quantum mechanic is as non-local as any other physical theory can be.