Abstracts for 2 hour workshops

Monday July 14

10:30-12:30: Workshops 2 hours Theme 3

Michela Maschietto (Italy) Room A303: Approaching conic sections with mathematical machines at secondary school.

In secondary and tertiary educations, students study conic sections mainly as an algebraic objects with a graphical representation. They rarely meet conic sections from a synthetic point of view. In addition, the origin of conics - as curves obtained by cutting a cone with a plane - has become a sort of “simple story” to introduce them, but this is not always picked up by the teachers during the lessons. In Italy, the recent reform of secondary school education requests to develop a synthetic approach to geometry.

This workshop aims to discuss the main steps of a teaching experiment focusing on the introduction of conic sections at secondary school level (16-17 years-old students) following the methodology of mathematics laboratory with mathematical machines (Maschietto & Martignone 2008, Maschietto & Bartolini 2011). In this teaching experiment the historical dimension is very important, because each mathematical machine has a strong link with the history of mathematics (Bartolini Bussi, 2005). In particular, we have considered mathematical machines with tightened threads and articulated antiparallelograms (described in Van Schooten’s books), big models of cones cut by a plan representing Apollonius’s theory and big models showing Dandelin’s theorem (all are available at the Laboratory of Mathematical Machines in Modena, www.mmlab.unimore).

The workshop is organised in steps as follow:

1. Introduction to mathematics laboratory with mathematical machines and to the context of the teaching experiment;
2. Working group on the analysis of worksheets for students concerning a first mathematical machine;
3. Collective discussion;
4. Working group on a second mathematical machines;
5. Collective discussion and historical perspectives;
6. Working group on a third mathematical machines;
7. Presentation of the final step of the teaching experiment.

References


Maschietto, M. & Martignone, F. (2008). Activities with the mathematical machines: pantographs and curve drawers. In E. Barbin, N. Stehlikova and C. Tzanakis (Eds.), History and Epistemology in Mathematics Education: Proceedings of the fifth European Summer University. Prague:
The use of history of mathematics has been included in Turkish middle school mathematics curricula with 2005 curriculum reform (Ministry of National Education [MoNE], 2005) and the following revisions (MoNE, 2009, 2013). The recent revised curricula state that history of mathematics can change students’ attitudes towards mathematics and its learning, it may make the classes more meaningful, and it shows mathematics as a cultural heritage (MoNE, 2013). However, the integration of these ideas into the sample teaching activities and formal textbooks is limited to the historical information from secondary sources such as biographies, dates, and names (MoNE, 2009, 2010) which are capable of serving only illumination approaches (Jankvist, 2009). Similarly, content and pedagogy of the elective course ‘History of Mathematics’ for pre-service middle school mathematics teachers is not effective in developing the pre-service teachers’ conceptions about using history in mathematics education as the course results in conceptions regarding illumination approaches (Alpaslan & Haser, 2012).

One effective way of using history of mathematics is to employ original sources. Jahnke (2000) addresses that such sources will have three distinctive benefits for both learners and teachers: (i) replacement of the typical perceptions about mathematics; (ii) reorientation of our ideas about mathematical concepts through making the familiar unfamiliar; and (iii) cultural understanding that deals with the evolutional process of mathematics linked with technological and scientific developments within the milieu of various societies. Through these practices, the original sources can catalyze learners’ comprehension of mathematical concepts within their linked historical structures (Knoebel, Laubenbacher, Lodder, & Pengelley, 2007). Moreover, possible benefits and restrictions of contemporary mathematical forms can be noticed by students and teachers through original sources (Tzanakis & Arcavi, 2000). Therefore, broad use of original sources seems to have a potential to address all aspects of whys and hows of using history in mathematics education (Jankvist, 2009).

We have determined to select and prepare the original sources as an initial step of the dissertation which will investigate the effects of using original sources of mathematics in enhancing pre-service middle school mathematics teachers’ conceptions about the use of history in mathematics education. In order to select the sources, we aim to set certain criteria within this study. First, the mathematical content in the original source should be within the scope of the middle school mathematics curricula (MoNE, 2013) (proper content and level of mathematics). Some learning objectives in the curricula can be achieved through the mathematical ideas in the original source (applicable). The original source should be interesting to study such as a source from the learners’ own culture (engaging) and so on. As for the preparation, the language of the sources, how to present them to be more meaningful, and what mathematical ideas can be inferred from them for the pre-service teachers’ and their future students’ mathematical
understanding will be mentioned. The existing models for employing original sources such as genetic and hermeneutic principles (Glaubitz, 2011) will also be considered. The determined sources will be analyzed based on the addressed criteria for selection and preparation.


Adriano Demattè (Italy) Room A212: Yes, I do use history of mathematics in my class because...

- First part of the workshop. Originals for didactics of arithmetic and algebra

In connection with my plenary, some texts from Italian mathematicians like Pacioli, Ghligai, or Tartaglia (originals and English translations) will be submitted to participants who will be asked to analyse the texts in order to plan some activities for students aged 14-16 years (structured laboratories or short exercises as well).

- Second part of the workshop. An open panel

The Man Keung Siu’s article “No, I don’t use history of mathematics in my class. Why?” will be briefly illustrated. Participants will be requested to write their reasons to use historical material in math classes (preferably if they really use it).
For people who won’t participate in the workshop, a large box with a poster like the following could be shown during ESU7 activities (near the plenaries room entrance, for example):

“Yes, I do use history of mathematics in my class because…”
Ten years after the “No, I don’t use history of mathematics in my class. Why?” - Observation and thought of school teachers collected by Man Keung Siu (Uppsala 2004).
Please, write in the answer sheet [copies will be available close to the box] and put it into the box.
If you prefer, use the e-mail address dematte.adriano@vivoscuola.it
[Also a paper copy of the Siu’s article could be shown]

Every answer could take place inside an article of the ESU7 Proceedings in which, at the same time, an analysis with respect to students’ lack of motivation in learning both history and mathematics will be carried on. An underlying question will lead the reflection: which ones of those ‘becauses’ could profitably inspire our school lessons?

Kristian Danielsen and Henrik Kragh Sørensen (Denmark) A214: Using authentic sources in teaching logistic growth: A narrative design perspective.

In Danish upper secondary education, history of mathematics is – and has been for decades – a mandatory part of teaching mathematics. Part of the reasoning is that history of mathematics is important in and by itself and that history of mathematics contributes to the students’ understanding of mathematics. In order to implement these ambitions, two challenges therefore arise: How to educate the teachers and how to devise suitable material for classroom usage.

Historians and educators of mathematics have addressed these two questions for decades, yet in this presentation we present a new perspective on the design of teaching material. Based in our teaching of a university course in the history of mathematics aimed at future teachers, we have developed teaching material about logistic growth. Central to the material is a short authentic mathematical source, namely Pierre-François Verhulst’s article from 1838 in which he introduced the logistic curve. Although short, that source invites discussions of four important aspects of the use of history in upper-secondary mathematics education: teaching a central mathematical topic, illustrating and discussing the mathematical modeling process, addressing issues of philosophy of mathematics, and informing the students about the historical complexity of mathematics. Our material includes a translation of the main source as well as a structured narrative providing the students with a window into the historical context and an understanding of Verhulst’s thought process.

In the workshop, we will briefly survey the use of historical sources in mathematics education. Then we will present and discuss our material and report on our experiences from using it in class. We then argue that this material could be used as a model for developing other new teaching materials and that it can serve as a standard for what a good teaching material should include.

Some references:
Frank Swetz (USA) Room A130: Pantas’ Cabinet of Mathematical Wonders: Collecting Materials for Convergence.

Since 2004, I have been collecting images of historical materials possessing a mathematical significance. Some of these images have appeared in the e-journal Convergence, sponsored by the American Mathematical Association of America. Mainly in the feature “Mathematical Treasures” whose purpose is to provide such images for classroom use and teaching reference as well as drawing attention to their possible research potential. This presentation will introduce the audience to a variety of images and materials and discuss their use and impact in mathematics teaching. Some specific teaching strategies will be introduced and, hopefully, audience discussions and interactions stimulated. References will be given to various existing collections of materials scattered among universities, special archives and libraries. As one university student exclaimed “Wow! I studied about Newton but now I have the chance to see his actual books”.

Bjarne Toft (Denmark) Room A104: Games of Piet Hein & Tribute to Martin Gardner.

The workshop will exhibit some of the mathematical games of the Danish poet and designer Piet Hein, in particular SOMA and HEX, and material related to them. The games are remarkable because of their simplicity of rules, in contrast to their difficulty of play. They can be enjoyed at many different levels and by all ages. The games were made famous worldwide by Piet Hein’s friend Martin Gardner, who wrote about them in his Scientific American column. Martin Gardner occupies a special place in twentieth-century mathematics, inspiring young and old, amateurs and professionals, to think about mathematical problems and have fun. His writings are culturally broad, clear and contagious. He made recreational mathematics a respectable discipline and argued for its use in education. His close friend, Persi Diaconis, wrote about him: Warning: Martin Gardner has turned dozens of innocent youngsters into math professors and thousands of math professors into innocent youngsters. The workshop will pay tribute to Martin Gardner in the centennial year of his birth.

Tuesday July 15

10:30-12:30: Workshops 2 hours: Theme 2 in six parallel sessions
In 2009, algorithmics was explicitly introduced in the new mathematics curriculum for the first year of secondary education in France. This introduction was extended to the new curricula for the second and third year published in 2010 and 2011. In the latter, the intentions of the curricula developers regarding algorithms are clearly stated: algorithms should be part of a problem-solving approach integrated in the other topics of the mathematical curriculum (analysis, geometry, statistics and probability, logic) and they could also be connected to other disciplines.

Algorithmics is therefore not meant as an independent sub-part but as a spiral work throughout the high school mathematics curriculum. Having this in mind, we had the idea to integrate history of mathematics in this approach to algorithmics. The following article presents two classroom activities based on the reading of original sources and experimented with first and third year students in two different classroom contexts. The first activity is a computer-assisted exercise session meant as an introduction to the chapter on quadratic functions for first-year students and based on a problem by Al Khwarizmi. The second activity is a guided research session based on Heron’s method for the approximation of the square root of a number. It was intended for third year students enrolled in the scientific section (Terminale S) as a conclusion to the chapter on sequences and limits and was carried out in small groups. After describing the pedagogical intentions and conceptual process, we review the activities and summarize the pupils’ work. We end up with an assessment of these two classroom activities from both pupils’ and teacher’s standpoints. In particular we try and assess the relevance, in these two cases, of the use of historical material and of the introduction of a historical perspective in teaching mathematics.

Nitsa Movshovitz-Hadar and Batyia Amit (Israel) room A210: Opening the door to in-service teachers for interweaving “tomorrow’s history” in the teaching of high-school mathematics.

At ESU5, Prague 2007, we discussed the rationale for integrating mathematical news in high-school mathematics teaching, in order to cope with the problem of the increasing gap between contemporary mathematics and school mathematics (Movshovitz-Hadar 2008). The proposed pedagogy was interweaving mathematical-news-snapshots (abb. MNSs), on a regular basis, in the ordinary teaching of high-school mathematics, where each MNS would be a 15-20 minutes PowerPoint presentation devoted to a single piece of news published in the past decades including its (often long) historical background and (often rich) applications as well as (some) yet unsolved related problems. We advocated then an empirical study to examine this proposed pedagogy as a solution to the problem and its impact on students’ perception of mathematics as a long-living creative part of human culture.

A 3 years action-research in a few classes followed and took place during 2008-2011. Parts of this study and some of its results were presented at ESU6 workshop, Vienna 2010 (Amit and Movshovitz-Hadar, 2011).

As of 2012 a 3 years feasibility study has been supported by The Israel Science Foundation, and carried out in 2 public high-schools every year. The whole mathematics team in each school participated in a sequence of 90 minutes weekly workshops in which a specialist exposed the team to a series of MNSs, one at a time, and empowered them through specially
designed activities, (such as reverse-engineering tracing back the project goals in each MNS, as demonstrated at our ESU6 workshop) to integrate these MNSs, on a regular basis, in their ordinary teaching during the school year.

This proposal is focused on sharing with the audience of ESU7, the challenge, the difficulties and successes we obtained during this experimental work in one of the high-schools with its 7 mathematics teachers, who worked with us for 2 years, and exposed about 400 high school students to MNSs. (A Sample MNS can be found in our 2011 papers).

References:


Michel Roelens and Wilfred de Graaf (Belgium) room A212: Workshop on the Use and the Mathematics of the Astrolabe.

Background of the workshop:
The astrolabe workshop is based on an idea by Prof. Dr. Jan P. Hogendijk, University of Utrecht. The workshop has been held in recent years on several occasions in Iran, Tajikistan, The Netherlands, Turkey, Syria and United Kingdom by Wilfred de Graaf. In Belgium the workshop has been held by Michel Roelens. Recently a detailed instruction on the use of the astrolabe and in particular on the mathematical method of stereographic projection has been published by Michel and Wilfred in Uitwiskeling, the Belgian journal for high school teachers.

Abstract: For more than one thousand years the astrolabe was one of the most used astronomical instruments in both the Islamic World and Europe. Based on mathematical principles dating back to Greek antiquity, the astrolabe flourished in the Islamic World from the year 800 CE onwards. The astrolabe was used to locate and predict the positions of the sun and stars, for instance to schedule prayer times, and to determine the local time. In this workshop the participants will first learn how to use the astrolabe. The provided cardboard astrolabe model has been recalculated for the latitude of Aarhus. The model is based on the astrolabe constructed by the mathematician and astronomer Abu Mahmud Hamid ibn Khidr Khujandi in the year 985 CE at the observatory in Baghdad. The astrolabe of Khujandi is one of the oldest extant today. It is currently displayed at the Museum of Islamic Art in Doha, Qatar. The workshop participants will learn to do some basic calculations with the astrolabe. These are for example:
to calculate the length of daylight in hours and minutes of a particular day of the year
- to determine the local time (so using the astrolabe as a watch)
- to determine the direction of the sun (so using the astrolabe as a compass)

In the second part of the workshop the participants will learn about the mathematical and
astronomical principles on which the astrolabe is based. These are, for example, stereographic
projection, the ecliptic, the zodiacal signs and the celestial sphere. We will focus in this workshop
on stereographic projection. Stereographic projection is a mathematical method to project a
sphere on to a plane. It has the nice property that it projects lines and circles on to lines and circles,
and not on to ellipses or hyperboles, for example. Therefore, astrolabes can be constructed
relatively easily; in principle it can be done by only using a ruler and a compass. We will explain
the mathematical properties of stereographic projection and will show how the lines and circles on
the astrolabe can be computed.

We believe the astrolabe is a very powerful didactic instrument to learn on the one hand about the
movements of the Earth, the Sun and the stars, and on the other hand about the mathematics that
is behind the method of stereographic projection. Also, we believe, the astrolabe is a wonderful
historical tool to enthuse young students for the study of mathematics and the natural sciences.

Materials:
The workshop participants will be provided with
- a handout with background information on the astrolabe, instructions for the use of the
  astrolabe and exercises
- a cardboard astrolabe model. The model is based on the year 985 CE astrolabe of Abu
  Mahmud Hamid ibn Khidr Khujandi and is recalculated for the year 2000 and for the
  latitude of Aarhus (i.e. 56 degrees North)

Public: A general public, but in particular high school students and/or teachers and (first or second
year) university students.

Jan Van Maanen (Netherland) room A214: Telling mathematics' revisited.

The history of mathematics has a strong oral tradition. People tell each other problems and
methods, and not so much in classrooms but rather in coffeehouses and during walks and parties. I
studied this phenomenon before and reported about it at the European Summer University at
Louvain, calling it “Telling mathematics”. Interest in these problems continued, as can be seen from
the recent book Mathematical Expeditions - Word problems across the ages by Frank Swetz
(2012).

I will shed some new light on this culture, of passing mathematical problems and knowledge by
sharing it with others. In Louvain my focus was on the role that such problems and especially the
act of telling it to fellow students, could have in the classroom. In this presentation I will take a more
historical and anthropological point of view. An experiment with two groups of about 40
mathematics teachers each will provide information about the repertoire of professional
mathematicians, as far as ‘telling mathematics’ is concerned. And some of these problems I will
trace through history. Many of them originated in Asia, and entered Europe in the Middle Ages and
Renaissance. And they continue to be told.

An interesting didactical question arises, which is why this spontaneity of sharing problems with
each other is observed rather outside school. What can we, teachers, learn from that?

Caterina Vicentini (Italy) room A104: Playing with Euler. From original sources to games: an
original approach to mathematics

Since 2010 my students and I have animated the “Mathematical Games’ Corner” of the Scientific Communication Festival “Scienza under18” that will have his fifth edition from May the 8th to May the 10th 2014 in Monfalcone (GO) Italy. During the third edition (2012) we presented a new game which was born during a work on original sources examined during some extra-curricular workshops on History of Maths held during the school year 2011-12. The participants were all volunteers’ students (aged from 15 to 18 years) frequenting the ISIS “D’Annunzio-Fabiani” in Gorizia, coming from the linguistic, scientific and artistic sections of our Institute.

We have started from some arithmetical pages of Euler’s Algebra and constructed three different versions of a game for different pupils (from 6 to 11 years old) and students (from 12 to 19) on arithmetical progressions: easy, medium and difficult.

To play one needs a billboard, two dice and a person who asks the questions and controls the answers giving the right ones if they are wrong. The board has 28 cubbyholes. With the first thrown one simply enters the play, then, after a thrown the participant has to answer a question about progressions: if he answers correctly, he will advance the points obtained, otherwise he will advance three points less. The winner is the player that first overcomes the 28th slot.

During the workshop, we will examine the pages of Euler’s Algebra we have taken in account, we will play the difficult version of the game and have a look on the easier versions, and we will discuss about the transposition of the original source in a game as well as the value of social games in increasing of students’ motivation in the learning of mathematics. We will also consider the improvements of school mathematics results of weak students participating to this workshops and animating the “Mathematical Games’ Corner” of the Festival.

Robin Wilson (UK) room A130: Two introductory university courses on the history of mathematics: introducing ‘Early Mathematics’ (from Egypt, Mesopotamia and China)

Over the past six years I have been involved with the preparation and presentation of two courses at a basic level on the history of mathematics. The first was for interested adults, while the second was for liberal-arts college freshmen in North American Universities. Both courses were based around Marcus du Sautoy’s award-winning BBC-Open University television series ‘The Story of Maths’ and on extra material that I wrote to accompany this series.

The first course was an Open University 10-point Level 1 Course, designed to ‘teach the maths behind the programmes’ to adult students studying at home. This course used a 200-page OU booklet that I wrote, and has now been successfully presented a dozen times to a total of about 2000 students. I shall describe the course content and the motivation for producing this course, and also describe the results.

The second had two forms, and was presented as ‘total-immersion’ courses to first-year liberal-arts students in Western Canada and in Colorado, USA. Like the Open University
course, it was based on Marcus du Sautoy’s television programmes and my course
notes, but also on an illustrated book co-authored with R. Flood. Again, I shall describe
and analyse the results of teaching these courses.
In this workshop I shall present some of the material and assessment from these courses
and use them to introduce various topics from ‘early maths’ – notably the mathematics
of ancient Egypt, Mesopotamia and China. No previous knowledge of these topics is
assumed.

Friday July 18
10:30-12:30: Workshops 2 hours: **Theme 1 & 6 in six parallel sessions**

Kathleen Clark and Emmet Harrington (USA) **room A303: Deciphering Mathematical Doodlings of
the “Shoebox Collection” of the Paul A.M. Dirac Papers.**

Florida State University Libraries Special Collections and Archives hold the complete papers of Paul
Adrien Maurice Dirac (1902-1984). The collection contains over 190 linear feet of family papers,
student papers, photographs, professional papers, certificates, regalia, sound recordings, office
collections of books and journals, and administrative information pertaining to Dirac’s work and life.
A full finding aid for the collection can be found at

In this workshop we will share an overview of the digital scans of 72 documents initially identified
for study as part of a project to digitize and preserve the “shoebox papers” in the collection. We will
briefly discuss the process of making sense of the mathematical doodlings found on a variety of
scraps of papers (Dirac never threw any paper away!). The majority of the workshop time will be
spent on the reconstruction of two investigations found in Dirac’s doodlings: (1) a systematic
approach for solving polynomial equations of degree \( n = 3, 4, 5, \) and \( 6, \) and (2) a selection of
combinatoric problems (which we are currently trying to unravel). Finally, we connect Dirac’s
doodlings to well-known sources from the history of mathematics and their treatment of similar
problems.

As a brief introduction, Paul Adrien Maurice Dirac was a Professor of Physics at Florida State
University from 1972 until his death in 1984 (although different resources report different “start
dates”). Among other discoveries, he formulated the famous Dirac equation, which describes the
behavior of fermions, and he predicted the existence of antimatter. Dirac shared the Nobel Prize in
Physics for 1933 with Erwin Schrödinger, “for the discovery of new productive forms of atomic
theory.” Dirac is buried in Roselawn Cemetery, in Tallahassee, Florida: “It was his family’s wish that
he should rest where he left the world” (Pais, 1998, p. 28).

Whereas those familiar with Dirac’s work would readily connect him to his famous equation or his
shared Nobel Prize with Schrödinger (whom he referred to as “Schröd” in the “shoebox papers”), we
have approached the collection at FSU with an eye on the lookout for mathematical investigations.
The initial motivation to investigate the collection in this way was that Dirac was “wooed...to do a
full mathematics degree free of charge” at Cambridge beginning in September 1921. Furthermore,
we believe the time period of the identified doodlings is between 1921 and 1923, though due to
the scrap paper nature of the papers used in our study, almost none of the pages are dated.

Gérard Grimberg (Brazil) room A210: Reflections about changing the teaching of geometry in graduation.

In Brazil, and the federal University of Rio de Janeiro in particular, the courses of licence split the teaching of the geometry in two disciplines distinct including on the one hand two disciplines from Euclidean and non-Euclidean geometry presenting, an axiomatic and synthetic approach (in the tradition respectively of Hilbert and Boliai), on the other hand an analytical approach, primarily Affin euclidean space definite starting from a normalized vector space. This dichotomy is found besides in secondary education where one teaches one year the geometry according to implicit axiomatic and the cases of congruences and similarities of the triangles, and another year analytical geometry.

This division of course has historical reasons which go up with the opposition between geometry synthetic and analytical in all the 19 century until work of Klein and Poincaré. opposition which is found in the handbooks of the 19th century when synthetic geometry and analytical geometry constituted two separate disciplines.

To cure in this established fact requires a reflexion on two levels. 1) theoretical: how to present the essential notions of the two points of view so as to constitute only one discipline, 2) didactic: how to determine a unifying element which makes it possible to decline the concepts and their applications.

With regard to the theory, we think of finding a solution in the presentation of the model of affin space checking the axioms of incidence and order, and defined a norm checking the axioms of congruence. Another aspect of the vector calculus related to affin space closely connected is to allow barycentric calculation and to deal with by this skew the problems of incidence and to introduce the concept of convexity. The barycentric co-ordinates are besides a first example of homogeneous coordinates, true bridge worm the projective geometry.

At the didactic level, the essential leading element is the traditional presentation of the problems as those treated in the small book of Coxeter (Geometry revisited), or more recently few problems of Geometry revealed of Berger, all with the perspective defended today by Daniel Perrin in various writing and courses. The perspective is not only solve this problems in one way but also, that is the most important, to present a multiplicity of solution about every problem. This multiplicity show in the fact we are not divising mathematics in little bits (synthetic, analytic, vectorial) but we are teaching only one thing, mathematics.

Texts presented int this workshop:
Brazilian mathmatic programs of the secondary level.
Brazilian cursus of geometry at the federal University of Rio de Janeiro.
Some french and English texts books in 19th century.
Few texts of Daniel Perrin (in his site)
Few classic problems which any future teacher cannot ignore, and didactical interest of these problems.

Tanja Hamann (Germany) room A212: New Math - a complete failure?

As in many European countries “New Math” has been implemented in German primary schools and has been abolished only few years after. Nowadays New Math is said to have been a mostly complete failure. In order to work out why this is the case one needs to describe the events, developments and difficulties that prove as crucial influences on the process of the reform. Among the most relevant sources are schoolbooks, which can be viewed – within the bounds of possibility – as a source for what happened in the classrooms. Therefore it is the aim of the workshop to try out textbooks and additional materials in order to analyse their didactical ideas, possible chances as well as possible difficulties for young pupils handling them. The textbooks that are to be worked on are Wir lernen Mathematik by Walter Neunzig & Peter Sorger (eds. 1968 and 1971, additional material: Logic Blocks) and alef by Heinrich Bauersfeld et. al. (eds. 1969 and 1975, additional material: matema Begriffsspiel), for both titles the focus is on material for 1st grade (6-7 years old). The books are in German but as one idea of the protagonists of the reform was to enable learning mathematics independently from children’s language abilities knowledge of German is not required to work with the materials.

Helder Pinto (Portugal) room A214: Mathematical lessons in a newspaper of Porto in 1853 (primary education).

As vias férreas são a escola primária da indústria, a escola primária popular é a via férrea de toda a civilização.¹ (Castilho, 1864)

The Associação Industrial Portuense (AIP; Industrial Association of Porto) was founded on May 3, 1849 (approved by royal law only on August 26, 1852) in the city of Porto (Portugal), still subsisting today under the name Associação Empresarial de Portugal (Portuguese Entrepreneurial Association).

“(…) The Industrial Association of Porto aims to develop the national industry – instructing the industrial classes and particularly the industrial workers in elementary arithmetic, geometry, drawing, and mechanical, chemical and physical arts; and especially in the study of machines, equipment and processes, which are being successively invented or perfected so that the Portuguese industry can be stated at the same level as the most advanced nations (…)“

(Statutes of AIP; Chapter II, Article 4, 1852)

In fact, the industrial education was one of the first priorities of the association, having created the Industrial School of Porto almost immediately (December 30, 1852). Even before that, although primary education was not their main focus, on December 6 of that year a course of «reading and writing» opened, which was attended by 117 students. This group included 25 individuals who attended these classes in order to propagate this knowledge for several villages around the city of Porto. António Luís Soares, professor of Polytechnic Academy of Porto since 1836 (First discipline: Arithmetic, Elementary Geometry, Trigonometry and Elementary Algebra), was intended to address to these 25 teachers in order to “present some works on teaching of arithmetics”. Just a few

¹ The railways are the industry's primary school; popular primary school is the railroad of all civilization.
could “attend the invitation” and so the alternative was to publish such works in the Journal of AIP (biweekly newspaper whose first issue came out on August 15, 1852). Thus, for several numbers (usually once a month, from April to December of 1853; Fig. 1) arithmetic lessons for primary education were published. These were addressed to teachers who should reproduce it to their students.

These Arithmetics classes were divided into sections, namely:

1. Formation of the numbers (in which a study of the metric system is included and compared with the usual Portuguese measures) – Fig. 2.
2. The first arithmetic operations – Figs. 3 and 4.

After these, several tables of units conversions were also published (linear, area, capacity and weight) – Fig 5.

António Luís Soares, as an introduction of these classes, does a brief but interesting analysis of how was the primary teaching in Portugal (particularly in mathematics), as well as several considerations for the importance of propagate the basic math instruction, either for industry either to the trade affairs workers, two very important activities for the economy of the city of Porto at that time.

In this workshop, we will present these classes in detail and highlight some consequences of the fact that they were classes promoted by an industrial association (also describing this association in more detail). We intend also to show how this publication may be used in the classroom (primary source in the history of mathematics) and how it may be interesting from the perspective of consolidating very basic notions of the discipline of mathematics.

**Figures**

**JORNAL DA ASSOCIAÇÃO INDUSTRIAL PORTUENSE.**

**NUMERO 16.** **SEXTA FEIRA 1 DE ABRIL.** **ANNO 1853.**

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\begin{align*}
&\text{um dez} \\
&\text{dous dez} \\
&\text{tres dez} \\
&\text{dez dez}
\end{align*}
\]

Fig. 2 Formation of numbers between 10 and 100
Fig. 5 Unit Conversion table

Bibliography

- Carvalho, A. Scipião G.; A Matemática na Academia Politécnica do Pó rto, in O Ensino na Academia Politécnica, pp. 1-31; Universidade do Porto, Porto, 1937.
The History of Mathematics could be a powerful tool for mathematics teachers to improve their teaching, by offering the students a variety of ways to achieve mathematical concepts successfully. This fact was in some way recognized by the Catalan Government Department of Education, which in the academic year 2007-2008, introduced some compulsory elements of the history of mathematics into the curriculum for secondary education. The new Catalan mathematics curriculum for secondary schools, published in June 2007, contains notions of the historical genesis of relevant mathematical subjects within the syllabus. However, there is no indication to develop the content associated with these subjects and, additionally, it is said that the proposal are only examples and teachers can choose others.

In any case, whether teachers choose the contexts proposed in the curriculum, as if they choose others, they should design activities so that students attain the concepts involved. For drawing up this kind of activities, a careful study and analysis of relevant texts related to the historical evolution of mathematical concepts is needed, and for doing that, teachers must be trained in History of Mathematics.

In fact, the academic year 2009-2010 saw the inauguration of a Master’s degree launched at the universities for training pre-services teachers of mathematics. However this kind of courses is only for pre-service teachers and a course for in-service teachers is also necessary. For this reason, also in the academic year 2009-2010, an online course on the History of Science for in-service science teacher training was put into practice. This course was drawn up by historians of science belonging to the Catalan Society for the History of Science and Technology (SCHCT) under the name “Science and Technology through History”.

This contribution deals with the historical mathematics part of this online course. We analyze, through the syllabus and development of this digital course for training teachers of mathematics, the principal features of the implementation of the history of mathematics in mathematics teaching. The aim of these analyses is to bring new elements of reflection for helping in the design and implementation in the classroom of resources on History of Mathematics.


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Bjørn Smestad (Norway) room A130: *What should a mathematics teacher know?*

In mathematics education, one way to approach the question of “What should a mathematics teacher know?” is through the theory/ framework of Mathematical Knowledge for Teaching (MKT), based on the work of Deborah Ball and others. In most articles on MKT, history of mathematics is barely mentioned or not mentioned at all. However, there are exceptions pointing out that history of mathematics has a role in several – or indeed all – the subdomains of the MKT model. In my opinion, a further exploration of this can give important insights into the discussion on the role of history of mathematics in teacher education and in mathematics teaching, as well as to enhance the MKT theory with insights from the work on history of mathematics.

In this workshop, I will pick important didactical examples from the literature of the HPM group to work on, to illustrate and discuss the role of history of mathematics in the framework of MKT. Thus, the workshop is aimed at mathematics educators. Regulars at HPM/ESU conferences might find the examples familiar.

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13:30-15:30: Workshops 2 hours: *Theme 4 & 5* in five parallel sessions

Man Keung Siu (China) room A303: *A geometric problem on three circles in a triangle (the Malfatti Problem) - A thread through Japanese, European and Chinese mathematics.*

This is a story about a geometric problem on three circles in a triangle. It appeared in the latter part of the eighteenth century in Japan, also in the early part of the nineteenth century in Europe and was studied in the latter part of the nineteenth century in China. The problem is about three circles lying inside a given triangle, each of which touches two sides of the triangle and the two other circles. It has come to be known as the Malfatti Problem (while the original problem actually asked for three non-overlapping circles of maximal area lying inside a given triangle), although the problem was studied by the Japanese mathematician AJIMA Chokuyen (1732-1798) before the Italian mathematician Gian Francesco Malfatti (1731-1807) studied it. By itself the problem is just one of many interesting problems in geometry. The point to be emphasized in this presentation
is not so much on its mathematical elegance but on the different historical and cultural contexts in which the same problem was studied in those different periods in different countries for different purposes. To invite views and comments from the audience so as to benefit therefrom this presentation will be run as a workshop with the exposition interspersed with problems in a prepared worksheet that aim at arousing discussion among participants.

Ysette Weiss-Pidstrygach (Germany) room A210. Historical Mathematical Models in Teacher Education - workshop based on didactical pedagogical material.

The development and production of mathematical models was already in the 19th Century and early 20th Century used for the training of mathematics student teachers. Nowadays it belongs to the history of European science. The use of historical mathematical models and their digitization in the study of mathematics, in particular in teacher training, as well as in the development of new mathematical models in digital and other forms makes it possible to relate historical, technical, educational and information technology aspects to each other.

The workshop presents a learning environment that can be used in a seminar for the Master education of mathematics student teachers. The seminar interrelates content of lectures on elementary mathematics, history of mathematics, mathematics teaching methodology and computer algebra.

Prior to the workshop, a reading course takes place, which makes students familiar with conceptual foundations for using history of mathematics as a tool and as a goal as well as with the corresponding concrete examples of classroom practice from different countries and time periods.

The interdisciplinary study of historical mathematical models in the seminar is carried out by three types of contextualization. These are:

1. Mathematical models as a historical object of study,
2. Historical mathematical models as a mathematical object of study and as a visualization of historical mathematical contents,
3. Mathematical models as a source of inspiration for experimentation, varying and developing new models and visualizations.

The study of historical mathematical models from a socio-cultural perspective includes aspects such as:

- Teacher training before, during and after the German Meraner Reform
- Intuition and perception of mathematics in the context of educational values and norms,
- Mathematical models in the context of the discussion between pure and applied mathematics,
- Historical mathematical models and patriotic education.

In the investigation of historical mathematical models, we restrict ourselves to models, mechanical instruments and drawing tools related to plane curves. The concrete choice of specific models depends on the mathematical preparation of the participants of the seminar and includes

- Selected static models,
- Selected kinematic models,
- Selected models closely related to school curricula.

Understanding the background of most models of the Brill and Schilling collection is a mathematically very challenging task for students. However, the visual and tactile access when working with visualizations and real models fosters this understanding. This takes place in the realm
of the third contextualization while dealing with the illustrated mathematics and the development of further illustrations.

The sources used in this seminar on mathematical models are mainly in German language. For the workshop, however, we provide for the selected models excerpts of these sources in English. Furthermore, some classical models and examples of the model production of student teachers will be available for study, experimentation and discussion.


In the workshop, the solar cycle, seen from different latitudes and at different times of the year, will be studied and linked to calendars in use at various times and places, based on the solar cycle. The solar cycle will be simulated by the use of the free software GeoGebra. Participants are encouraged to bring their own computers and download GeoGebra in order to create their own solar-cycle programs. Such programs are based on trigonometric functions and therefore suit students aged approximately 15 to 18 years. Furthermore, a choice of currencies will be explored and their origins studied in order to reveal their connections to their original socio-economic environments. Lastly, numerals, numerical systems and counting in selected languages and societies will be studied with respect to their origins and their connections to societal activities. Participants are encouraged to contribute examples and ideas to the topics above and other related topics.


Dans le cadre du festival triennal Europalia-India (octobre 2013 – janvier 2014), nous avons construit (avec cinq collaborateurs) une exposition « Art et Savoir de l’Inde » consacrée à six aspects ponctuels du savoir indien : 1° la géométrie du rituel védique, 2° les techniques de calcul dans le système décimal positionnel indien, 3° l’astronomie de Savai Jai Singh II (1689-1743), 4° l’architecture du temple de Minakshi (Madurai), 5° la médecine ayurvédique, 6° les jeux originaires de l’Inde.

L’exposition est constituée de 18 panneaux illustrés (2mx1m), d’objets et de maquettes explicitant les six aspects. Après une visite de l’exposition, celle-ci sert de prétexte à des activités pédagogiques à plusieurs niveaux. En voici quelques exemples :

1° avec les élèves de fin de primaire, nous explorons la cosmologie védique pour déboucher sur des puzzles représentant l’autel védique, des constructions de carré et des comparaisons d’aires des figures (différentes formes de briques) qui en découlent,

2° avec les élèves de début de secondaire, nous abordons les énoncés indiens du « théorème de Pythagore », illustrés par des triples pythagoriques, ainsi que les circulature et quadratures indiennes et les approximations de π qu’elles sous-tendent,

3° avec les élèves de fin de secondaire, nous envisageons la construction du petit cadran solaire (laghu samrat) de Jaipur et les calculs trigonométriques et astronomiques qu’elle a nécessité.
4° avec les étudiants de l’enseignement supérieur, futurs professeurs de mathématiques, nous réfléchissons à la manière d’utiliser le contexte et les textes indiens pour susciter l’intérêt des élèves en leur présentant les mathématiques sous une forme plus « humaine ».
Bien entendu, d’autres activités que celles énumérées ci-dessus auront été abordées et nous en présenterons un petit choix à l’aide des panneaux, objets et traces pédagogiques appropriés.

Harm Jan Smid (Netherlands) room A130: A Mathematical Walk in “Museum Boerhaave”.

The Dutch National Museum for the History of Science and Medicine (Museum Boerhaave in Leiden) is the home of a large collection of instruments and objects from the history of science and medicine, the oldest dating from the 16th century. It also owns a collection of mathematical instruments and devices, such as an enormous quadrant from 1610, made by Willem Jansz. Blaeu. Museum Boerhaave wants to attract a broad range of visitors and organizes special exhibitions and programs for children of different ages and for school classes. Some years ago it started a project around its mathematical objects, wanting to attract more attention for these objects and to interest children for the history of mathematics and to show them the relations with cultural history and school mathematics.
This resulted in a mathematical walk in the museum. Participants receive a 20-pages tour guide on A3-format which guides them along the different mathematical objects of the permanent exhibition. The tour guide contains not only explanations about the objects, but also interesting facts and stories about the history connected with these objects, assignments that can be made with these objects, maps, plotting paper to work on etc. Since the children are obviously not supposed to work with the original objects, they are for the tour equipped with a suitcase containing (simplified) replica of the original objects, which can be used to do the assignments. The mathematical walk is intended for schoolchildren around the age of fifteen in the middle classes of secondary school, an age in which it is often difficult to interest them for mathematics. It is hoped that by showing them that mathematics can also be fun and has interesting history gives them a more positive attitude towards mathematics. So far the mathematical walk seems to be a success; the museum is now preparing a mathematical walk for primary school children and a centre where school classes can work on (historically based) problems, including hands on activities.
In the presentation we will give you a short introduction to the Museum Boerhaave and its collection of mathematical objects. We will take you on the mathematical walk through the museum, present the tour guide and the suit case with the replica’s and works on one of the assignments. At the end we will discuss the evaluation on the whole project and the plans and ideas of the museum to develop more educational activities for mathematics around its historical collections.