Perfect Secure Computation in Two Rounds

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Joint work with Zvika Brakerski and Rotem Tsabary

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The MPC Zoo

Online/offline MPC for dynamic functionalities

Semi-honest
Computational
2-party OT-channels

UC-secure
Arithmetic MPC
with CRS

Covert MPC

Maliciously-secure MPC
Client-Server Model

Perfectly-secure MPC
with Correlated randomness

Adaptively-secure
MPC with fairness

Today: Simplest Model (BGW,CCD)

- N parties
- point-to-point private channels
- passive adversary
- honest majority
- perfect security
  - unbounded adversary
  - unconditional

“the simplest nervous system is in certain jellyfish”

“Should have been discovered by the ancient Greeks…”

Ronald Cramer (3 days ago)

Today: Simplest Model (BGW,CCD)

- N parties
- point-to-point private channels
- passive adversary
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- perfect security
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"the simplest nervous system is in certain jellyfish"

Simple BUT:

- Useful starting point for more realistic adversaries
- Still quite a few **open problems**

Completeness Results

1988:

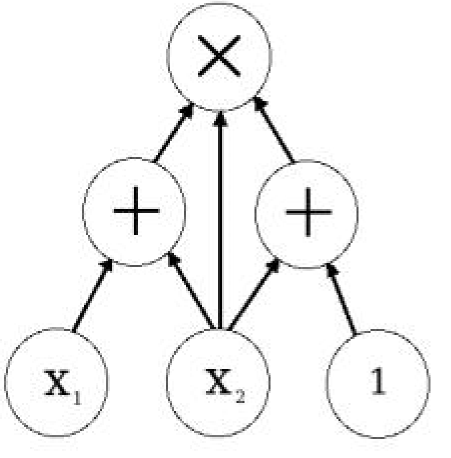
[Ben-Or, Goldwasser, Wigderson, Chaum, Crépeau, Damgård]

Thm. At the presence of honest majority, **every** function f can be perfectly computed

Tight: Honest Majority is necessary

Complexity: poly(circuit-size(f))

Rounds: Multiplicative depth of f



Constant Round Complexity?

[Bar-Ilan-Beaver-1989]: expected O(1) round for all f

- efficient protocol for NC1
- worst-case O(1) round with statistical security

90’s: restricted interaction patterns &
efficiency slightly beyond NC1

- [FKN ‘94,IK’ 97, CD ‘00]

Constant Round Complexity?

2000-02: [Ishai-Kushilevitz]

Thm. With honest majority,

3-round **perfect** protocol for all functions

- Randomizing Polynomials
  - Every function reduces to degree 3 computation

Constant Round Complexity?

2000-02: [Ishai-Kushilevitz]

Thm. With honest majority,

3-round **perfect** protocol for all functions

- Efficient for NC1 and log-space
- Computational variant for poly-size circuits [AIK05]

Constant Round Complexity?

2000-02: [Ishai-Kushilevitz]

Thm. With honest majority,

3-round **perfect** protocol for all functions

- 1 round is impossible
- Yields 2 rounds if privacy threshold <n/3

Open: With honest majority,

2-round perfect protocol for all functions?

- [IK00] cannot be achieved with degree-2 randomizing polynomials

Ishai-Kushilevitz 2000:

"An open question of a somewhat different flavor is that of finding the exact number of rounds required for privately evaluating an arbitrary (i.e., a worst-case) function f with an optimal privacy threshold.

Using randomizing polynomials, an upper bound of 3 was obtained. If this bound is tight (i.e., 2 rounds are not enough) then, in a very crude sense, the randomizing polynomials approach is non-restrictive."

Our Results

Thm 1: With honest majority,
2-round perfect protocol for all functions

- Efficient for NC1 and log-space
- New paradigm: Multiparty Randomized Encoding
  - Relaxes Randomized Encoding
  - Abstracts Garbled Protocols [Garg-Srinivasan-2017]

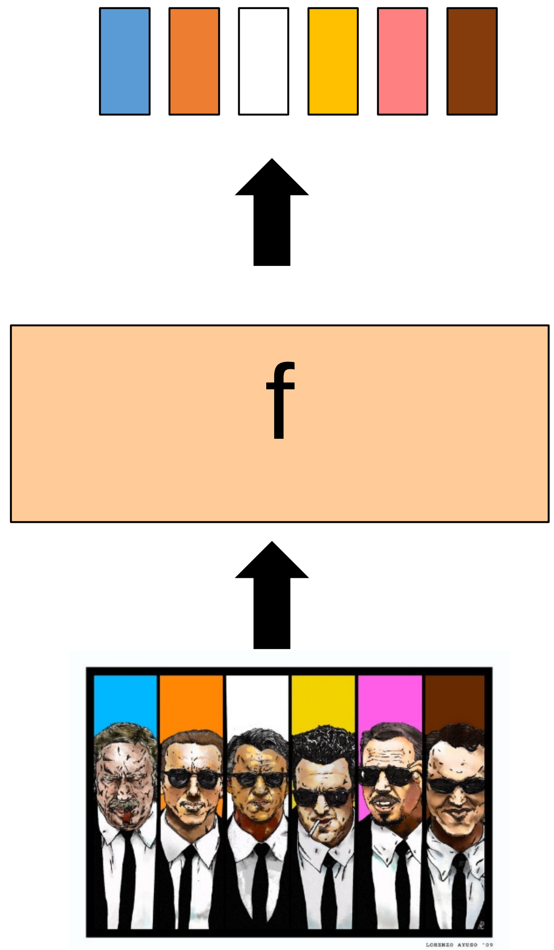
Thm 2: Assume OWF and honest majority.
eff. **2-round** comp. protocol for poly-size circuits

- Parties make only BB calls to OWF.
- Incomparable to [Garg-Srinivasan'18], [Benhamouda-Lin'18]
- We don't need OT but require honest majority

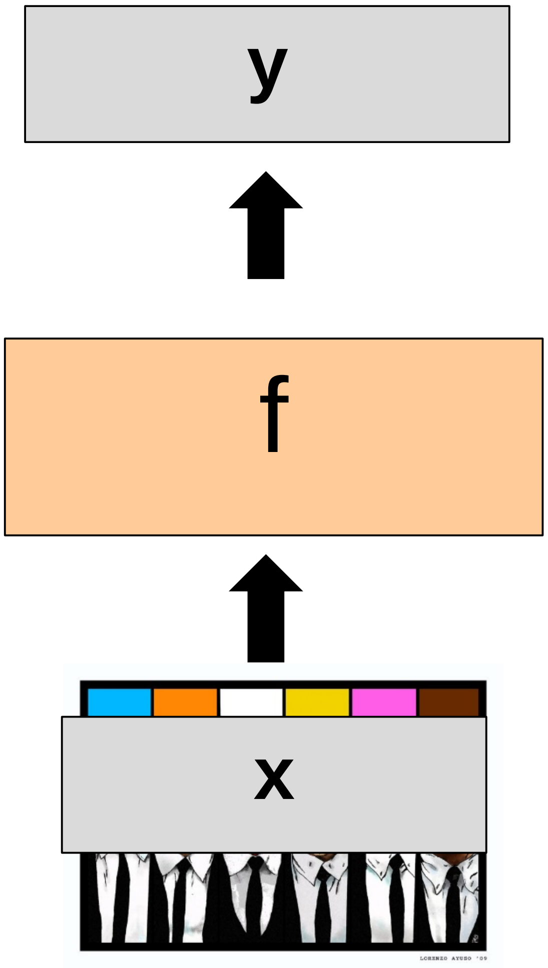
The rest of the talk

- Randomizing Polynomials
- Multiparty Randomized Encoding (MPRE)
- About the proof: MPRE with degree* 2
- Conclusion

Randomizing Polynomials [IK00]

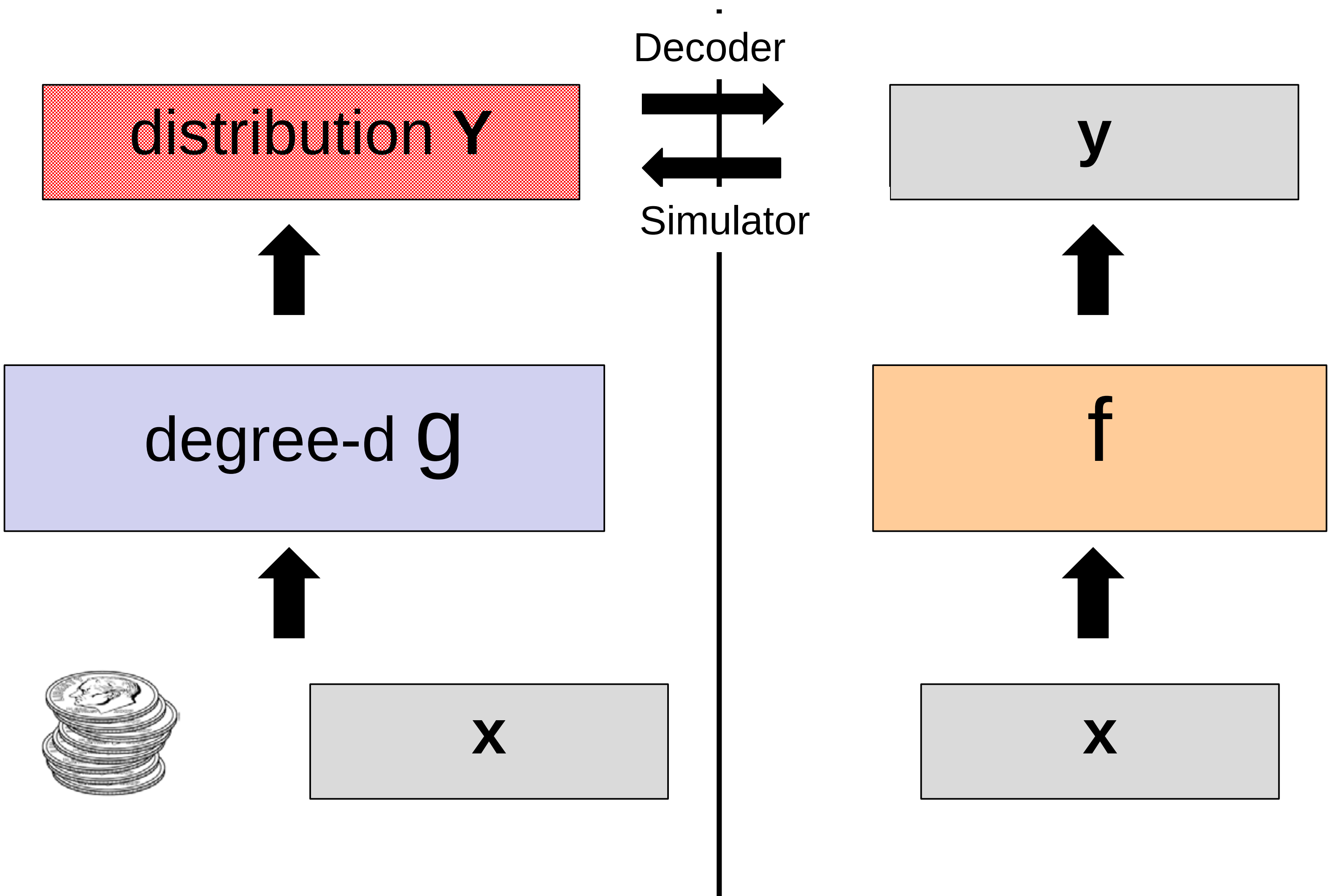


Randomizing Polynomials [IK00]



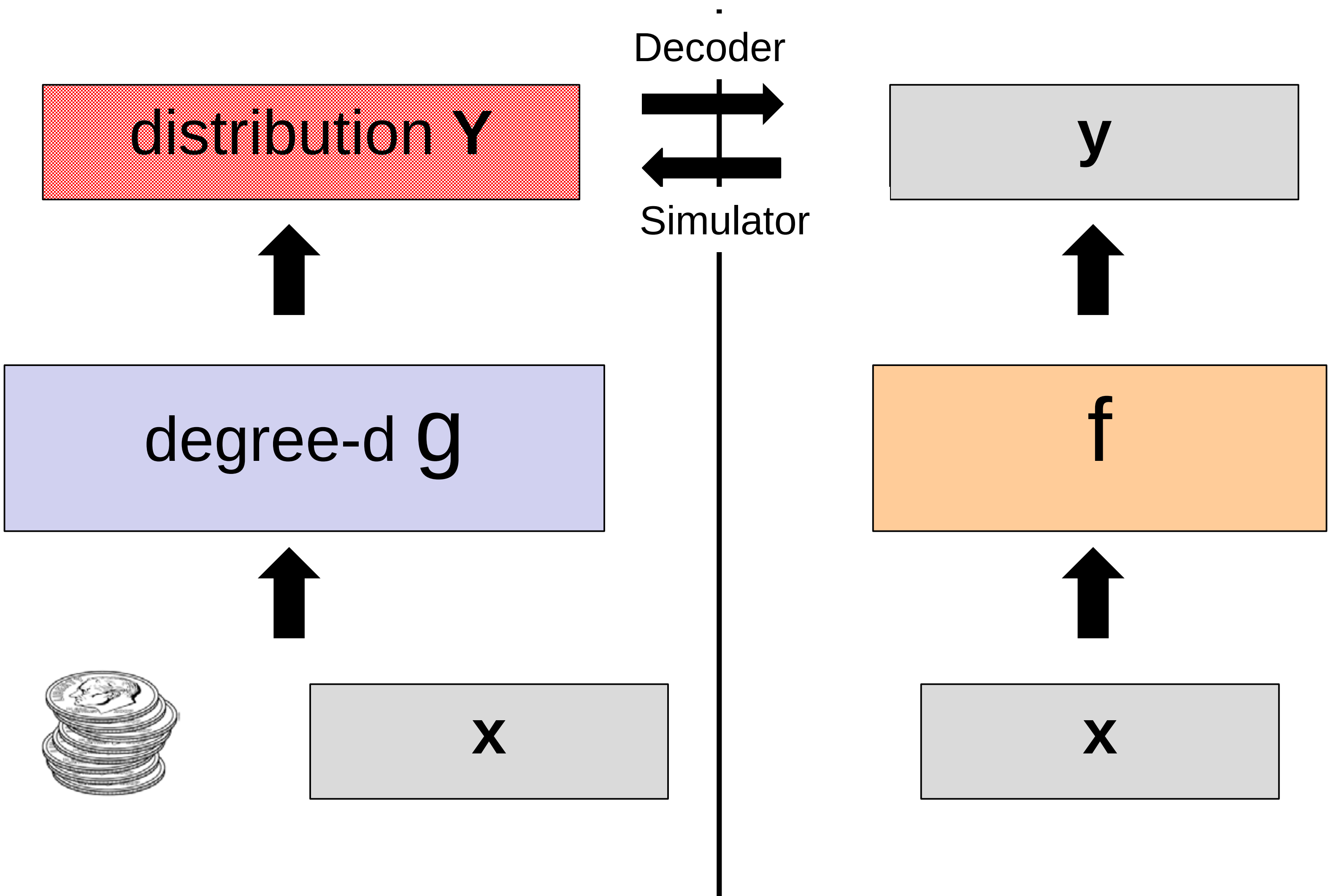
Randomizing Polynomials [IK00]

MPC for g $\Rightarrow$ MPC for f



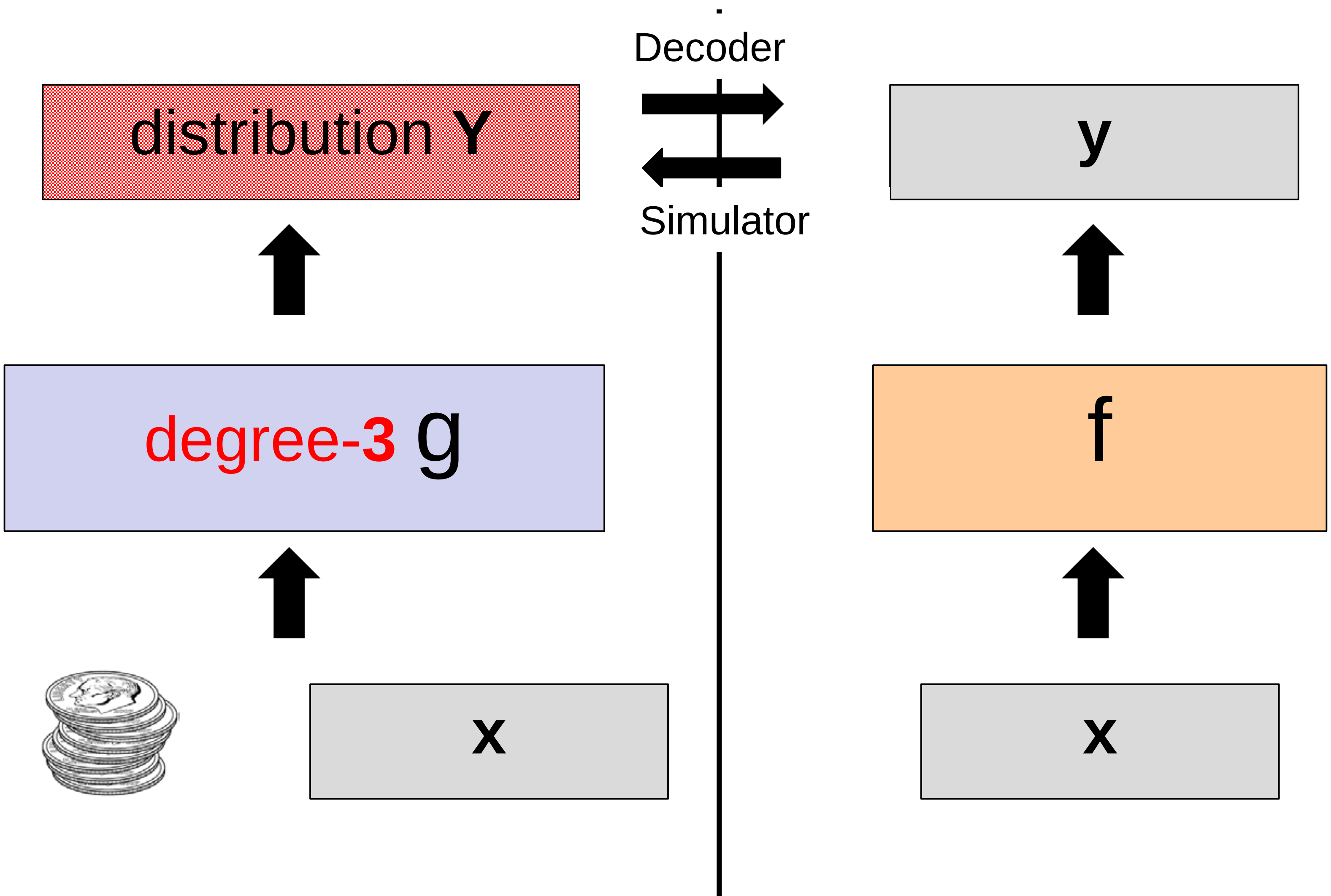
Randomizing Polynomials [IK00]

g has d-round protocol ⇒ f has d-round protocol !

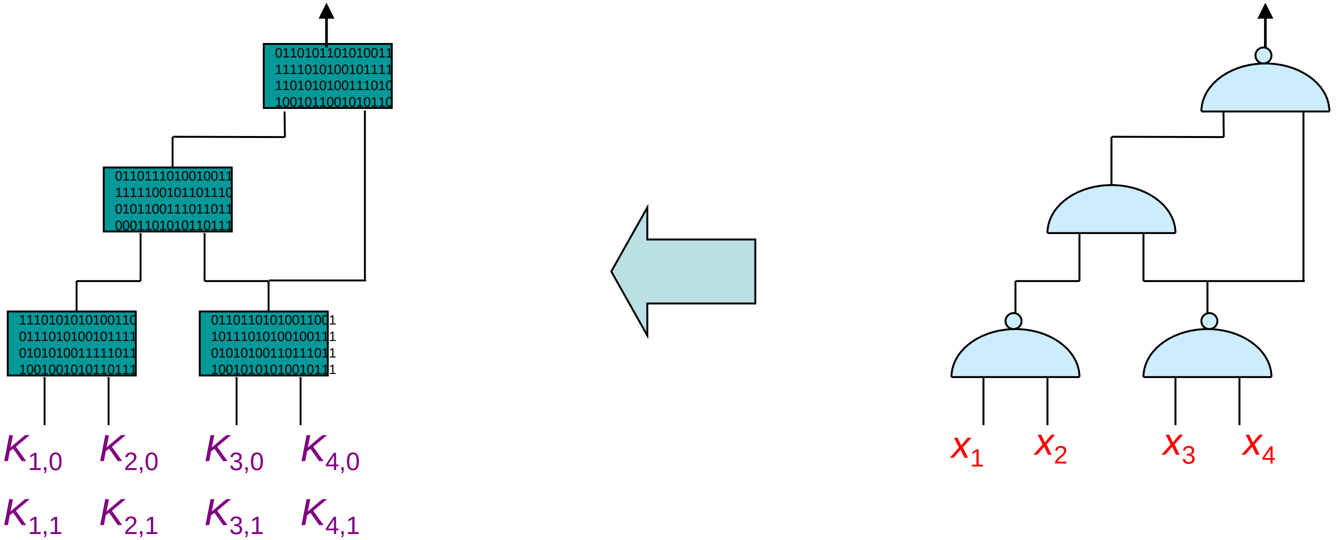


Randomizing Polynomials [IK00]

Thm [IK02] Every f has perfect RP of degree 3

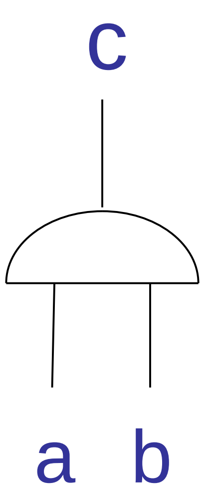


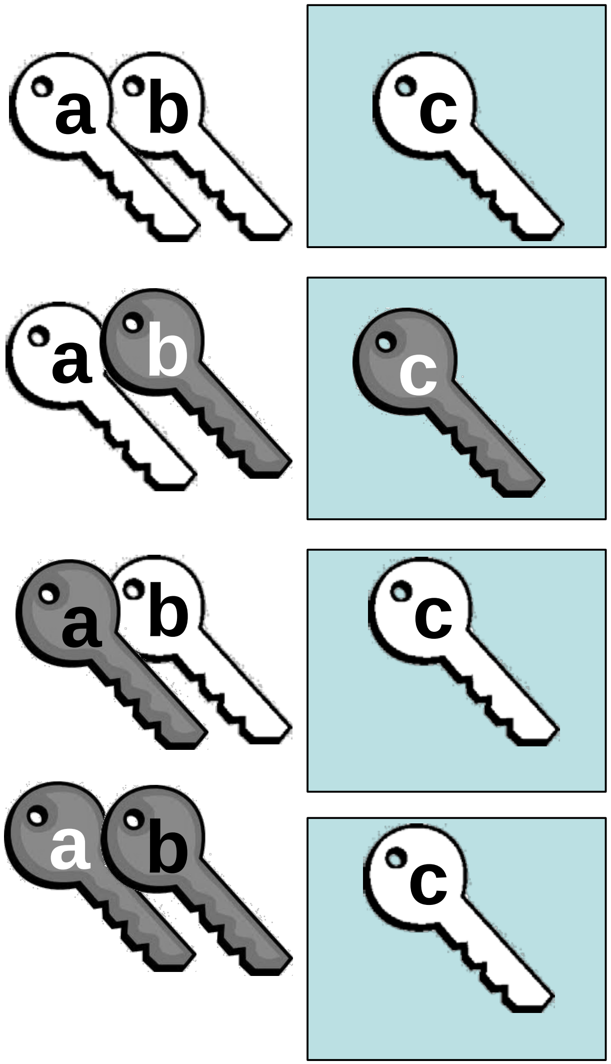
Degree-3 RP from Information-Theoretic Garbled Circuit [IK02]



$g(x,(k_{i,b},r)) = ((k_{i,x_i})_{i=1..n}$, garbled tables)

GC-based Randomized Encoding





Randomness per wire:

- random mask bit
- 2 keys

Release:

per input wire release:

- corresponding key

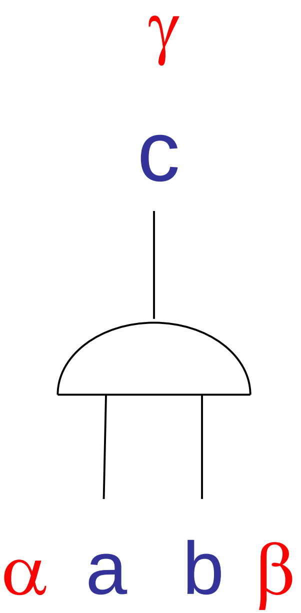
per gate:

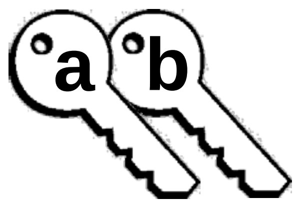
- 4 ciphertexts
- deg-3=deg(gate)+1

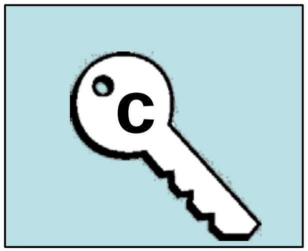
per output wire:

- release mask bit

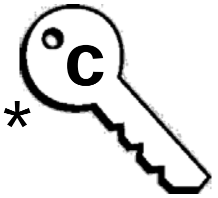
GC-based Randomized Encoding







$G(\alpha,\beta)*$



$+[1-G(\alpha,\beta)]*$

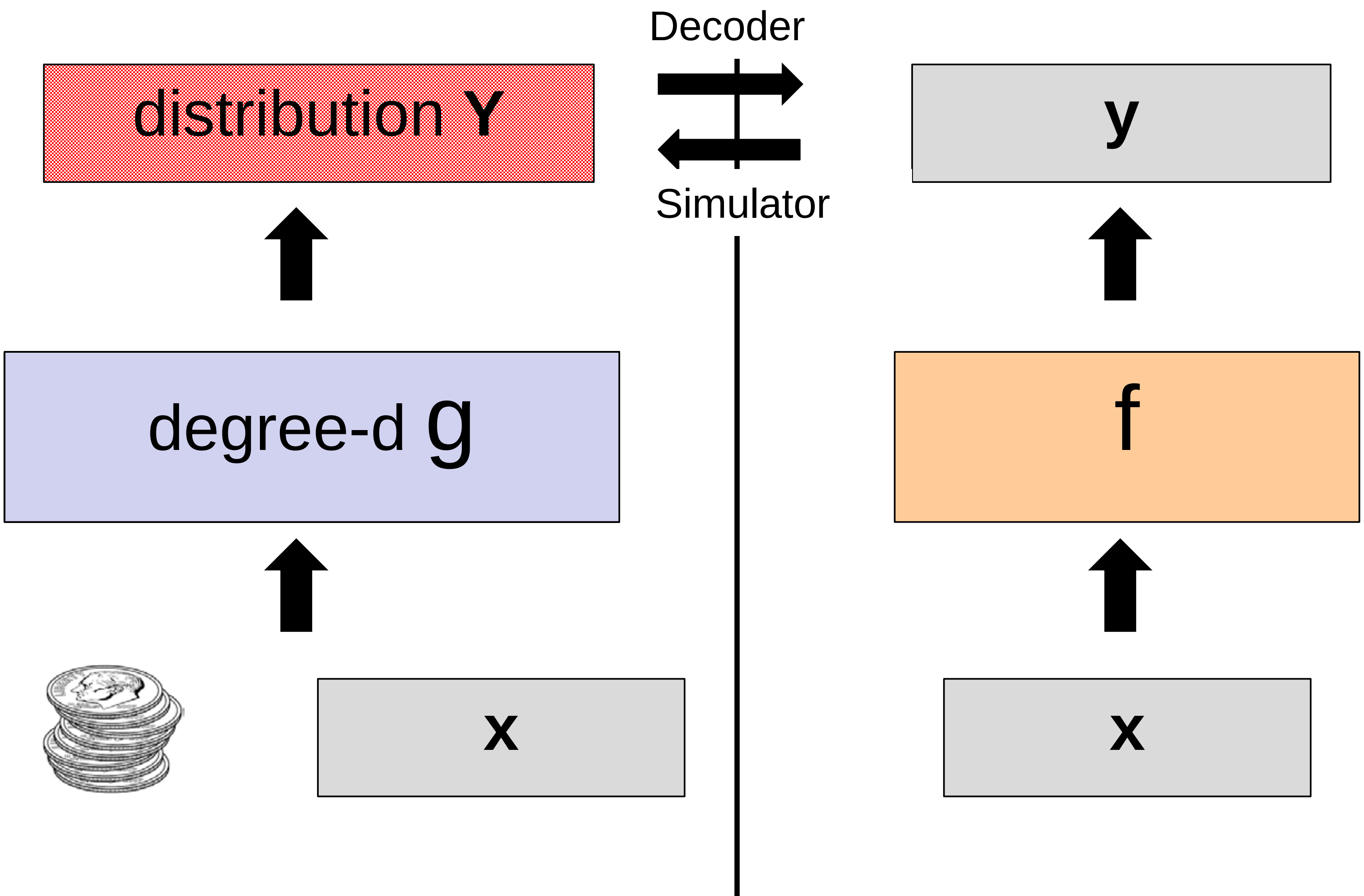
per gate:

- 4 ciphertexts
- deg-3=deg(gate)+1

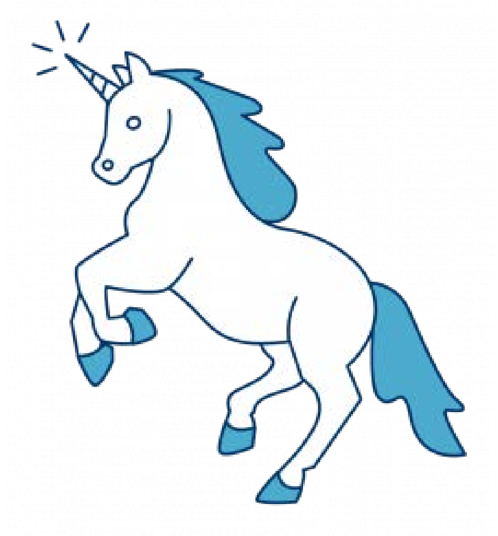
Randomizing Polynomials

Many other applications (e.g., parallel crypto)

- decouple simplicity from semantics !



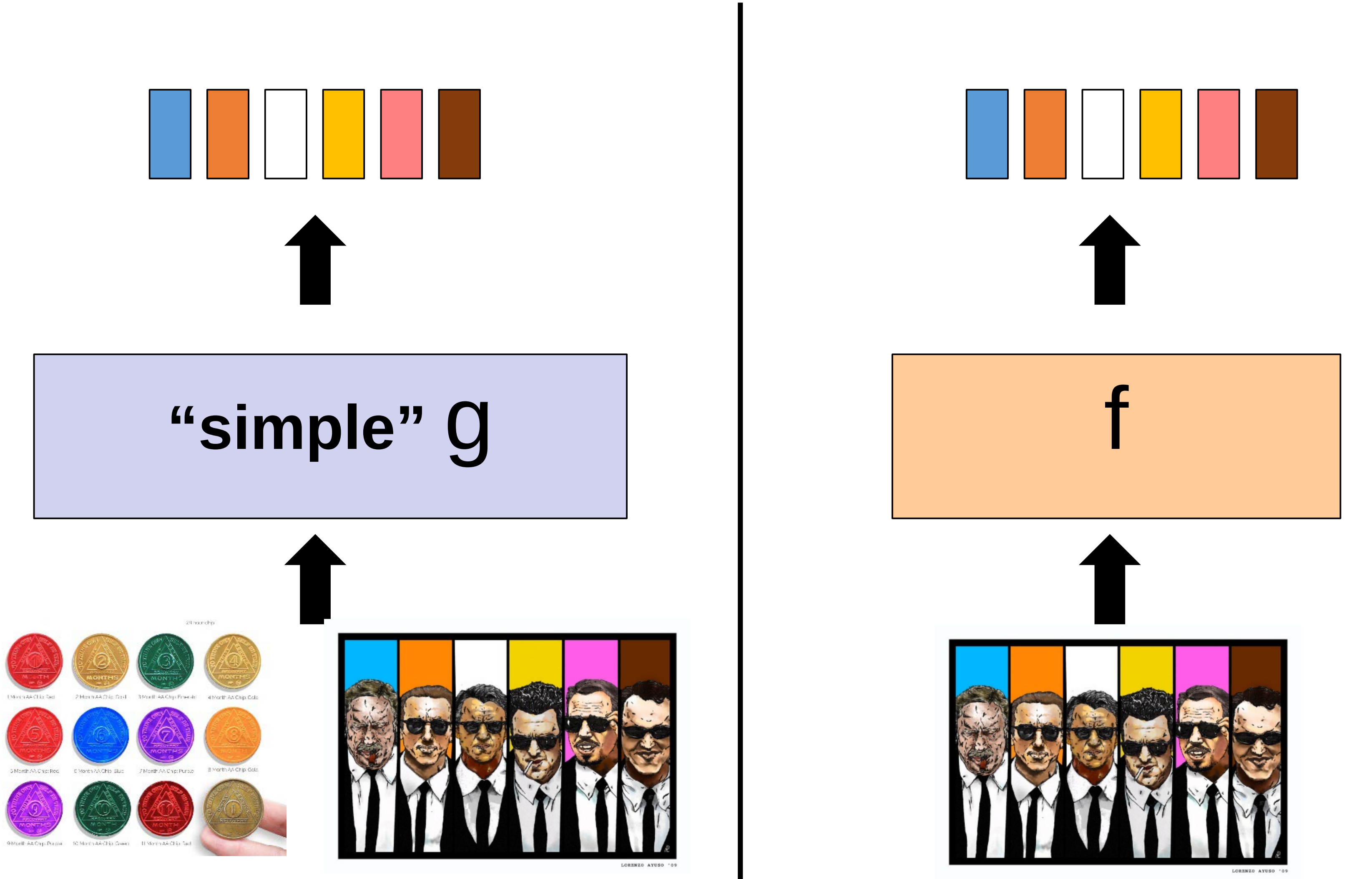
Problem:
For most functions,
NO degree-2 perfect RE's



Sol: Compromise!
Aim for a weaker notion

Multiparty Randomized Encoding (MPRE)

Relaxed correctness: Each party has a decoder



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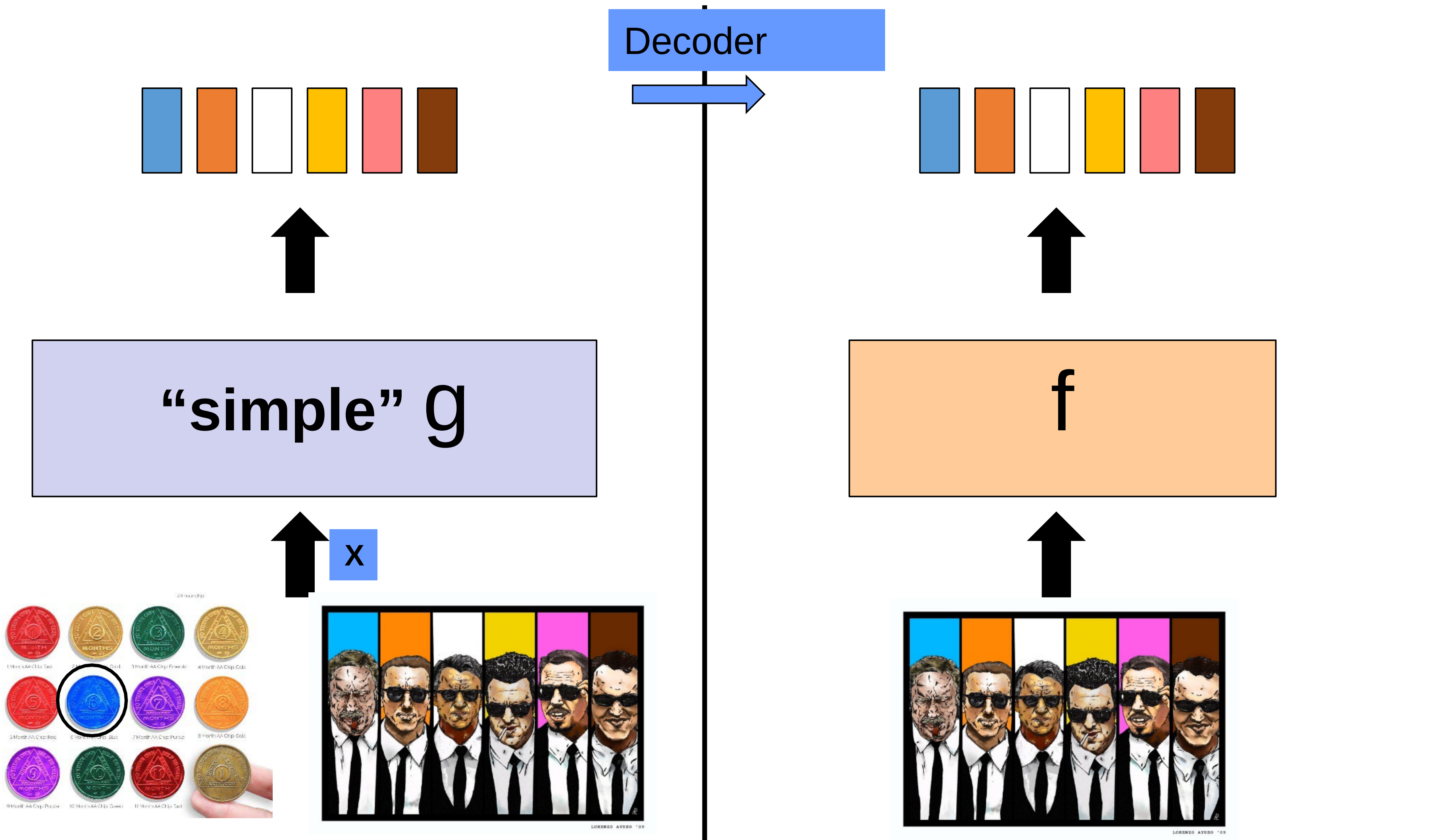


Multiparty Randomized Encoding (MPRE)

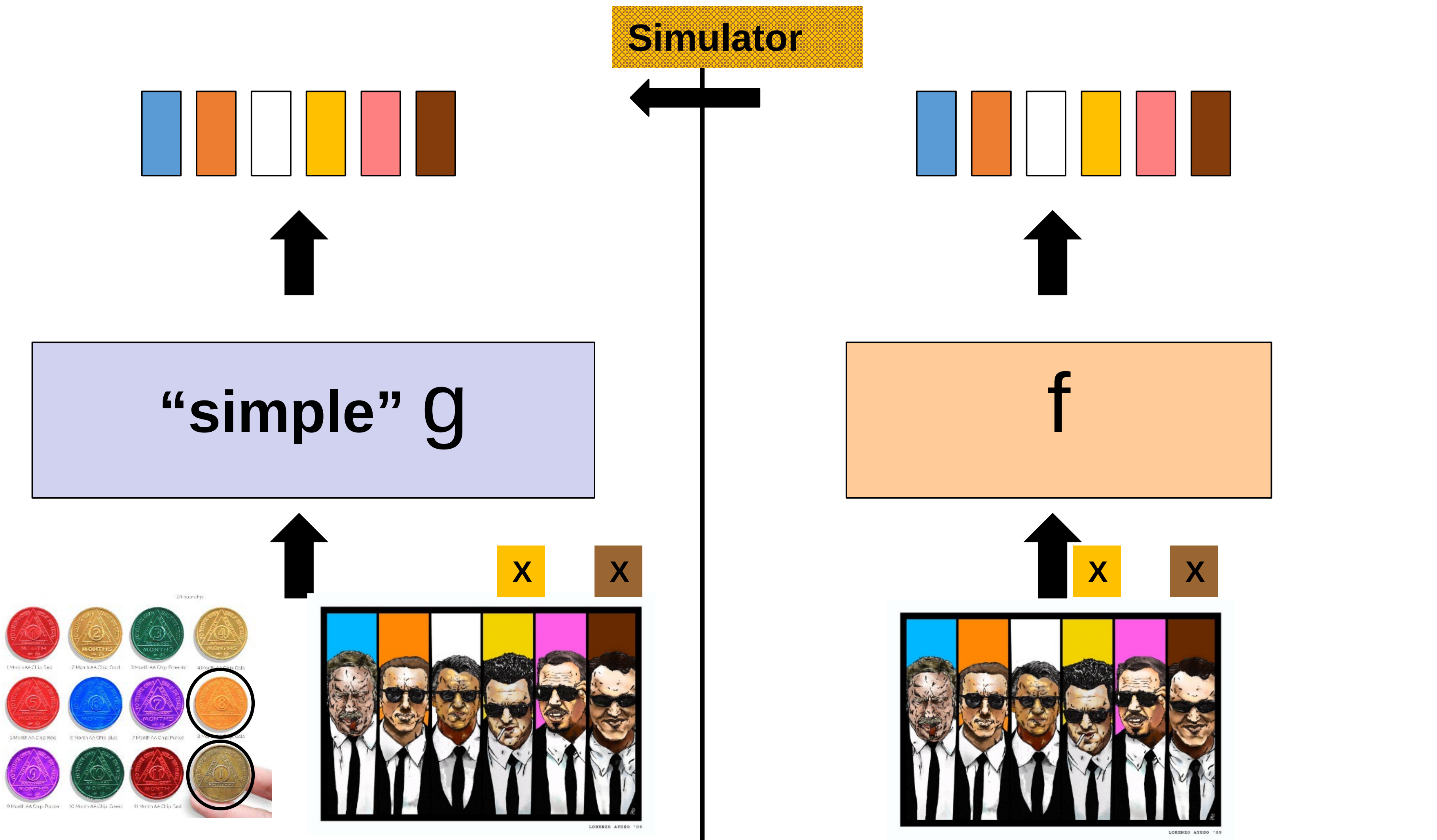
Relaxed privacy: Every minority has a simulator

“simple” g

f

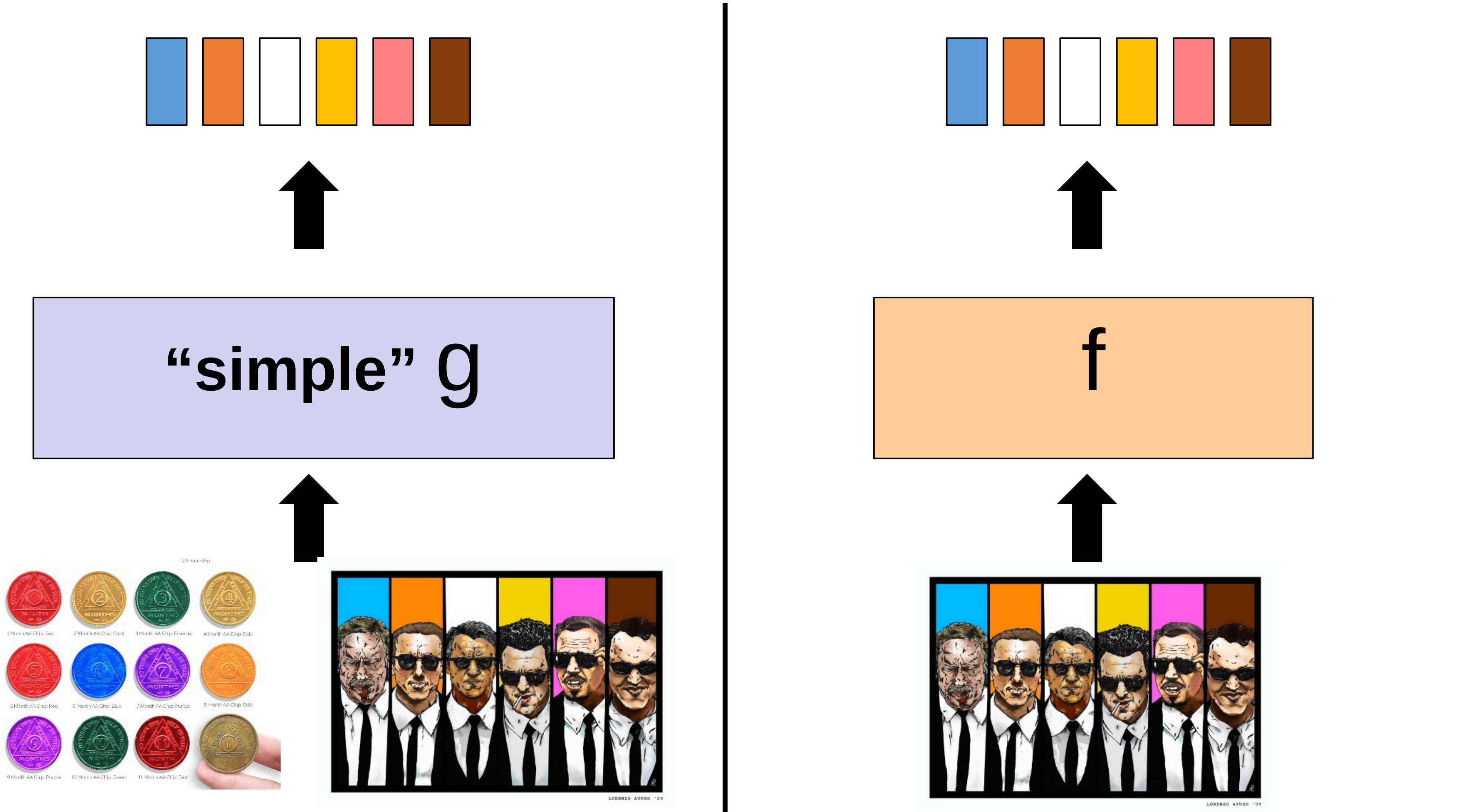
Multiparty Randomized Encoding (MPRE)

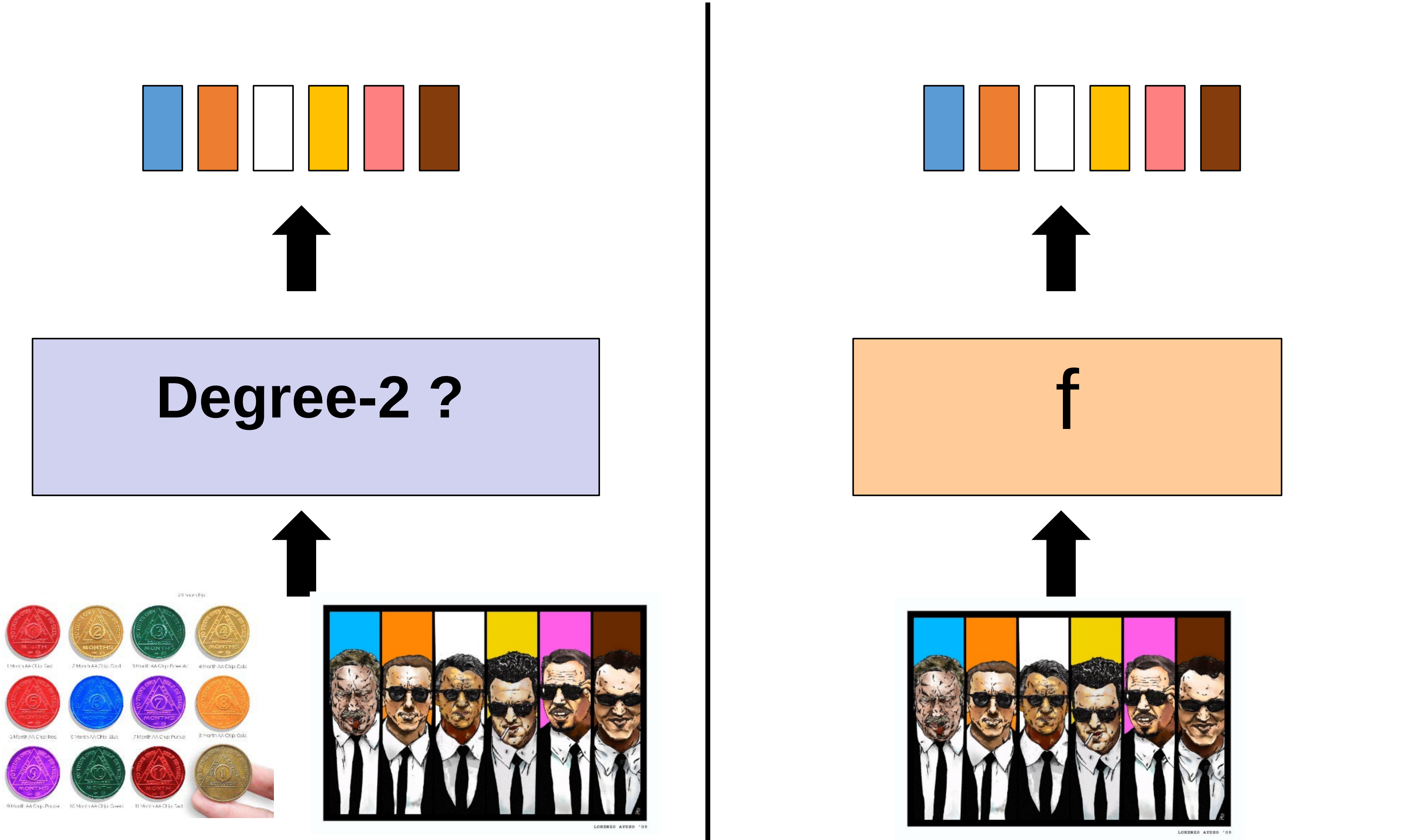
Relaxed privacy: Every minority has a simulator



MPRE **relaxes** Randomized Encoding

- Encodes functionality
- RE is a special case of MPRE
- Protocol for g $\Rightarrow$ Protocol for f



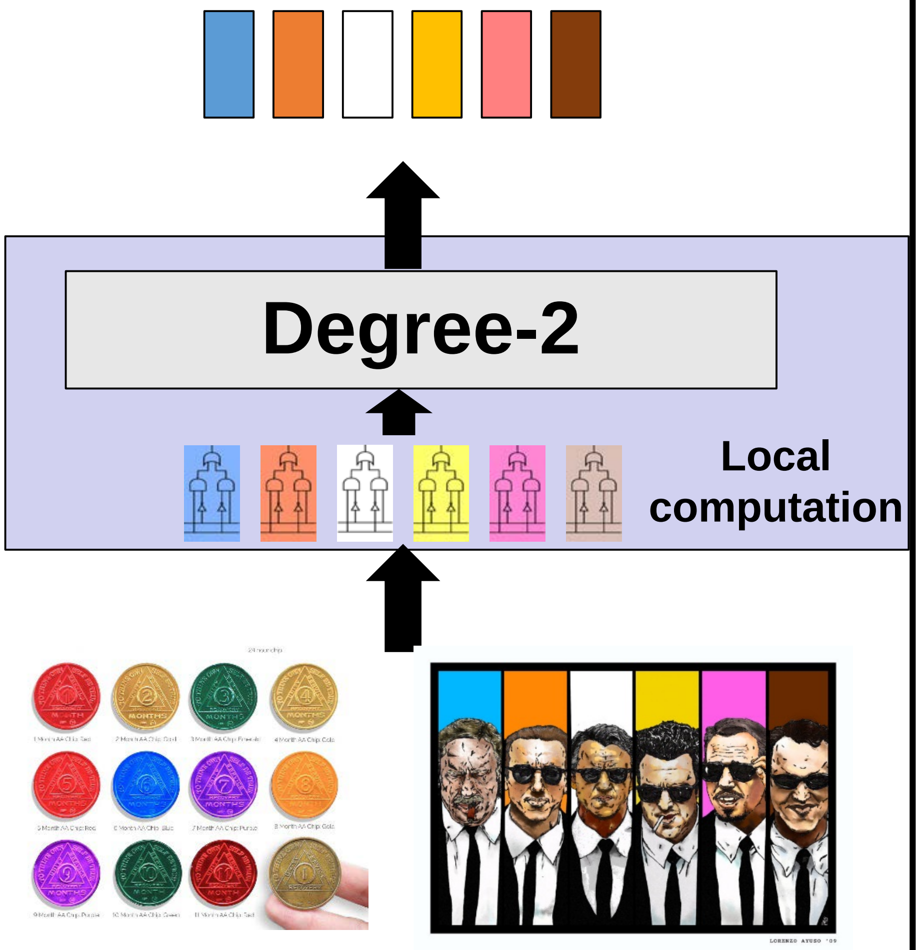


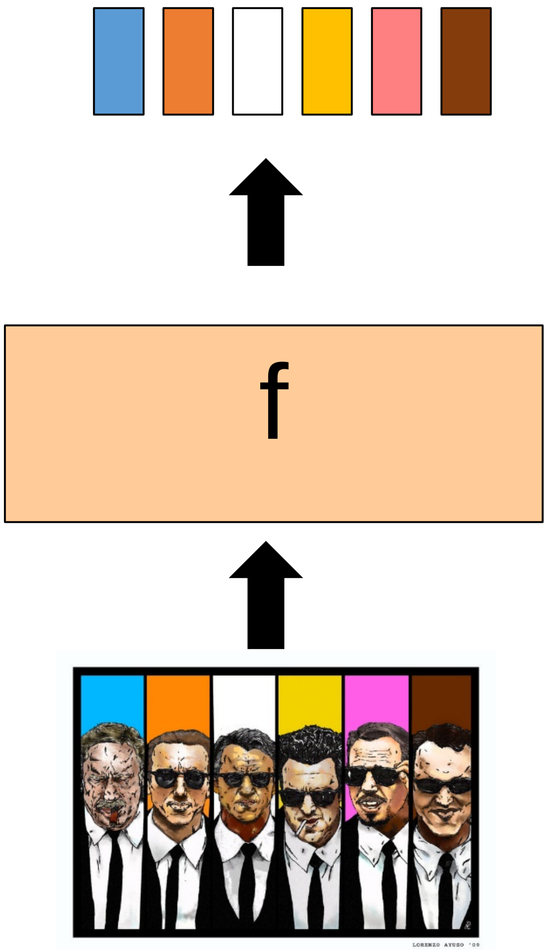
Degree-2 ?
f

Thm: every functionality has MPRE of “effective” deg-2

- Efficient for NC1
- Efficient computational-MPRE for general circuits

⇒ 2-round honest-majority protocol



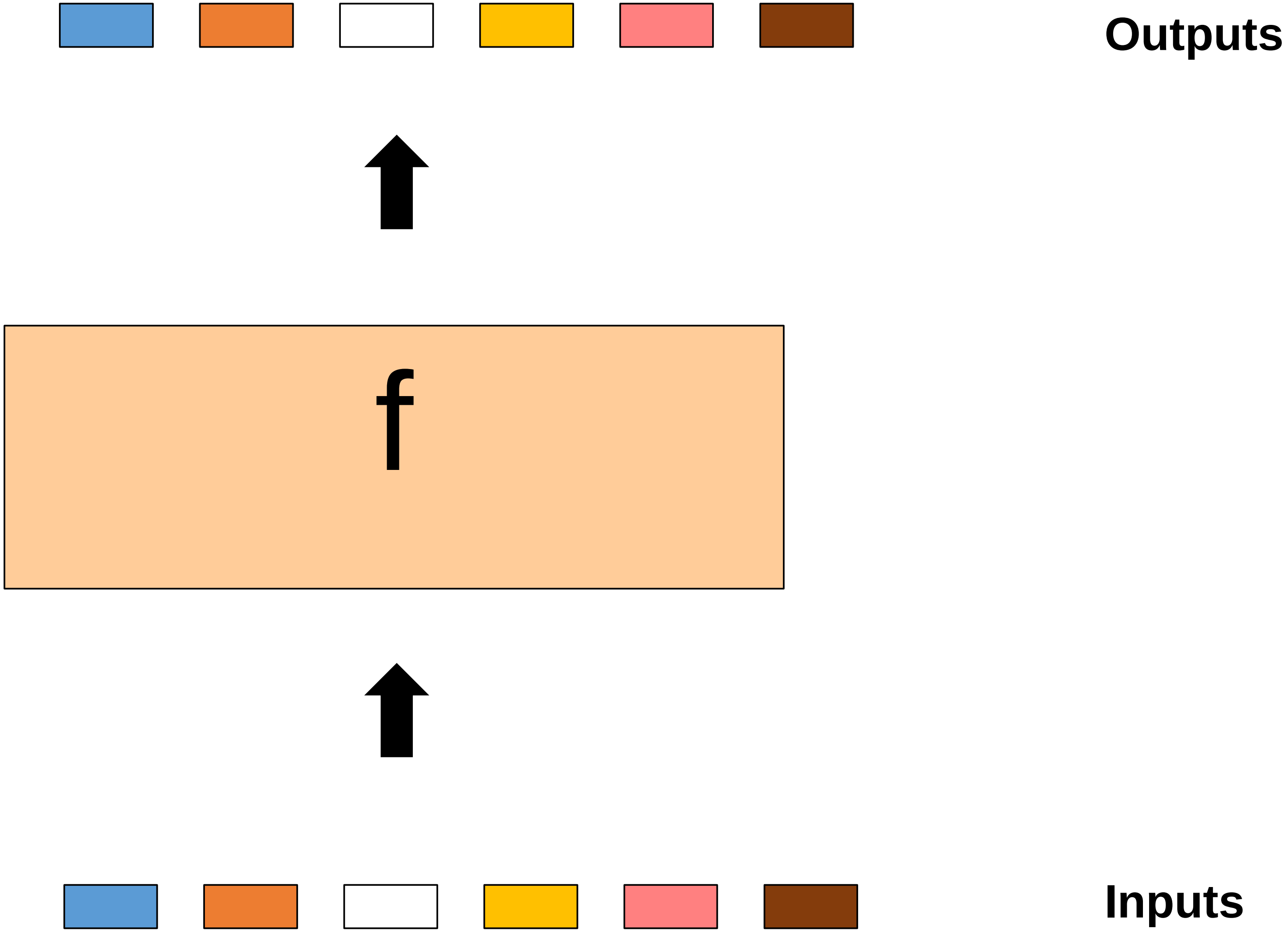


Proof Idea

1. From protocols to “nice”-MPRE

2. From “nice”-MPRE to deg-2 MPRE

Step 1: Protocol Induced MPRE



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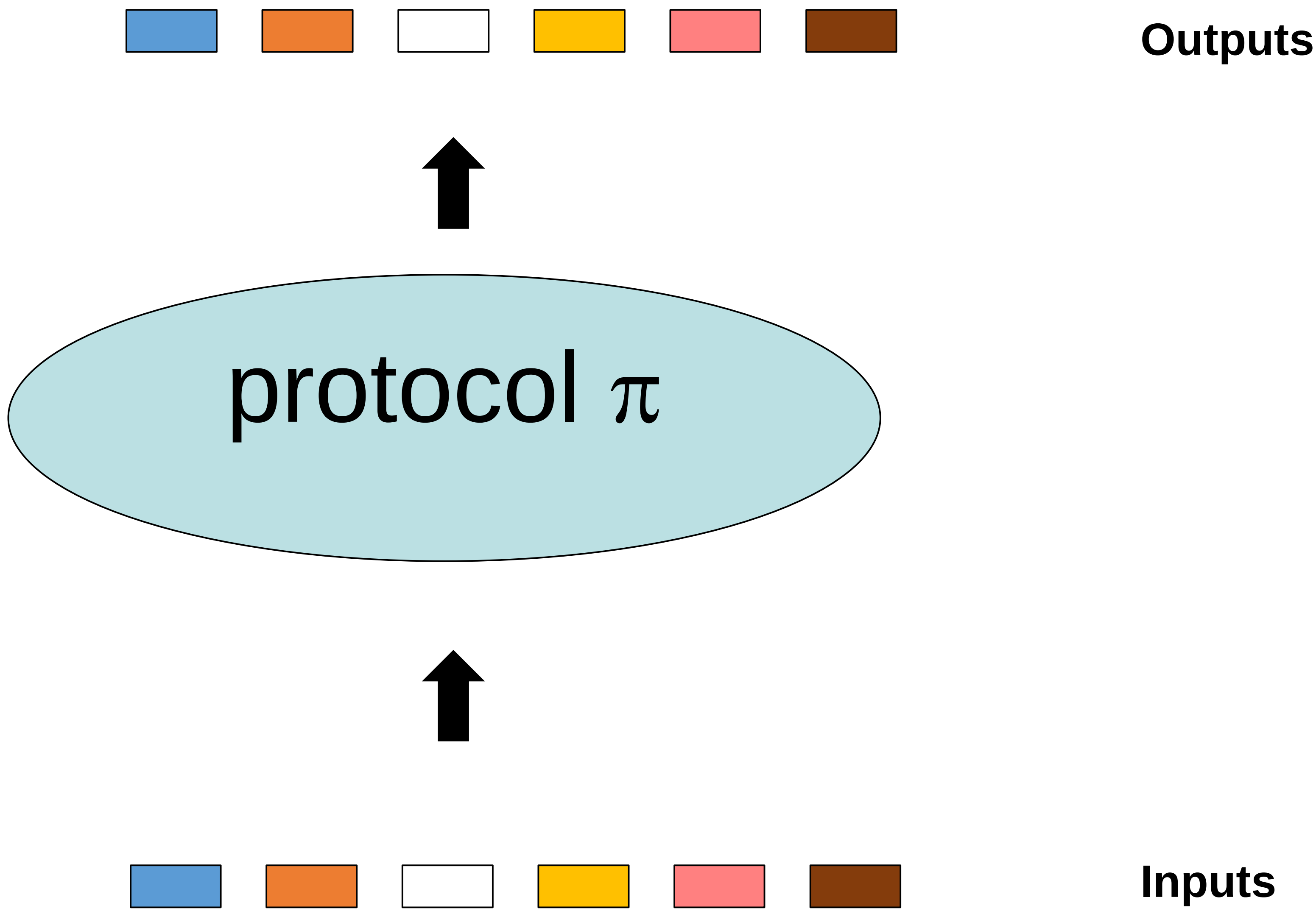

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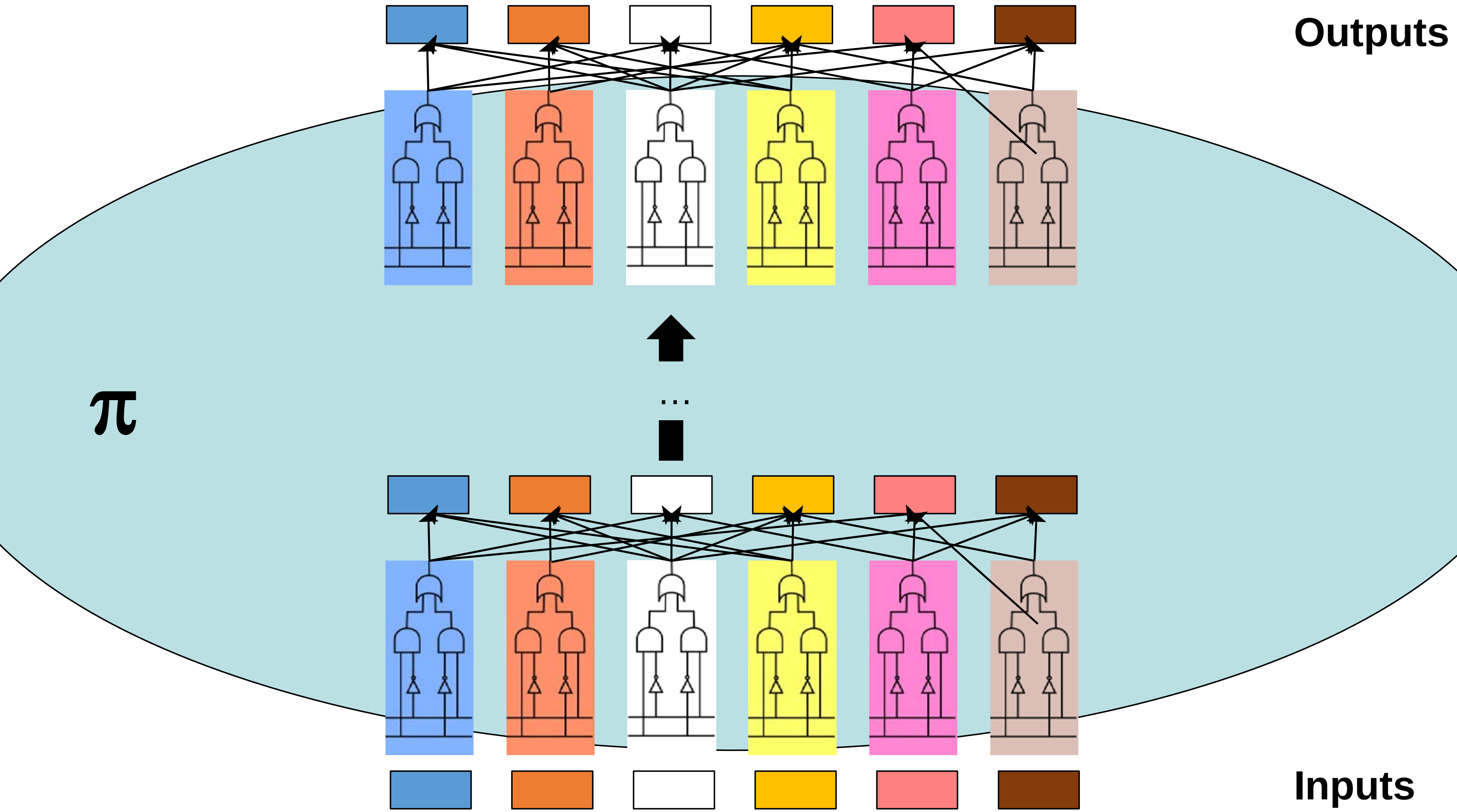
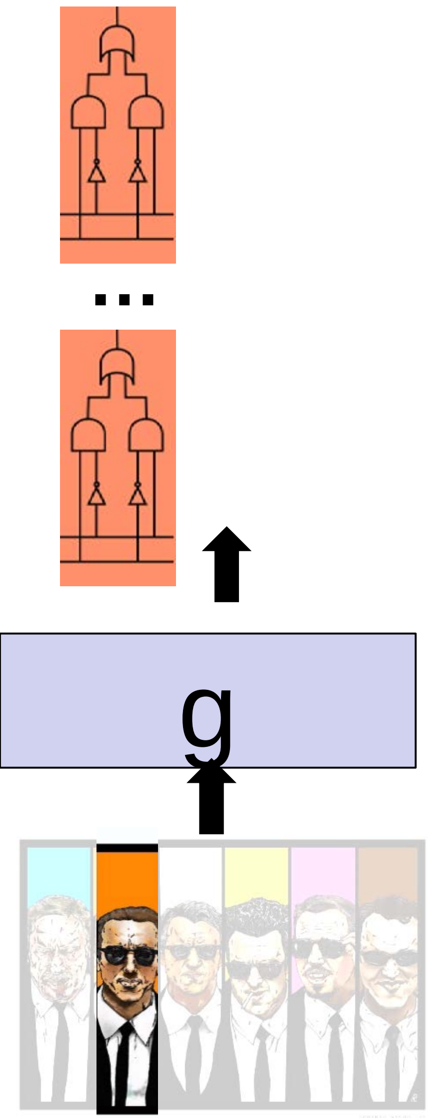
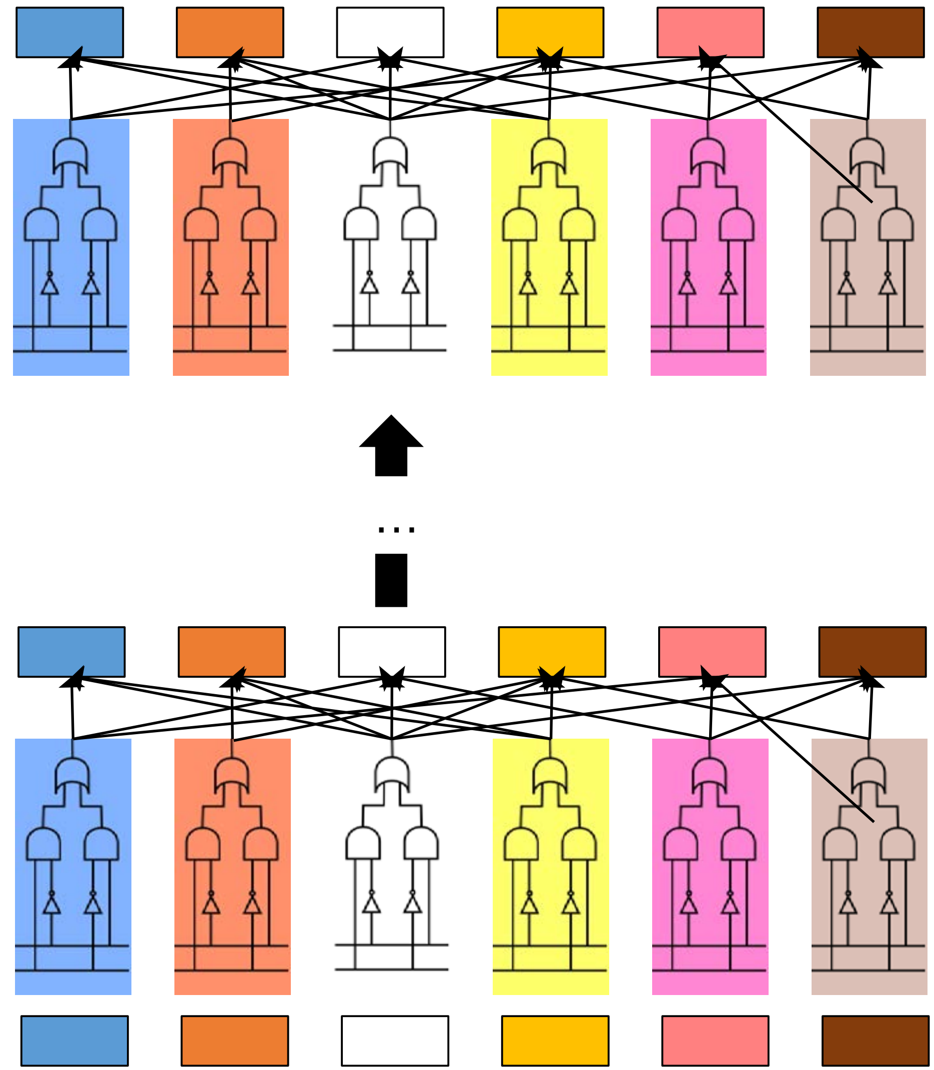


Step 1: Protocol Induced MPRE

Let g be MPRE that gives to a party its view & intermediate values



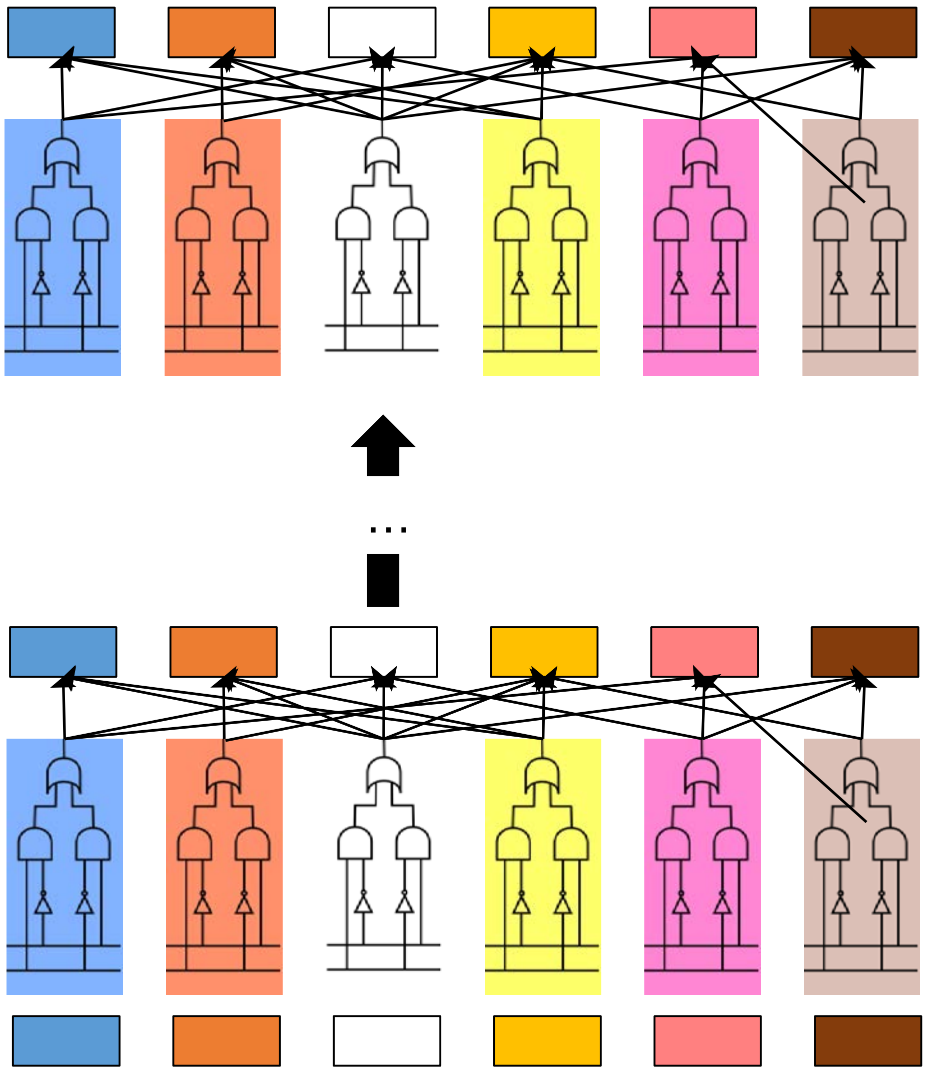


Key observation

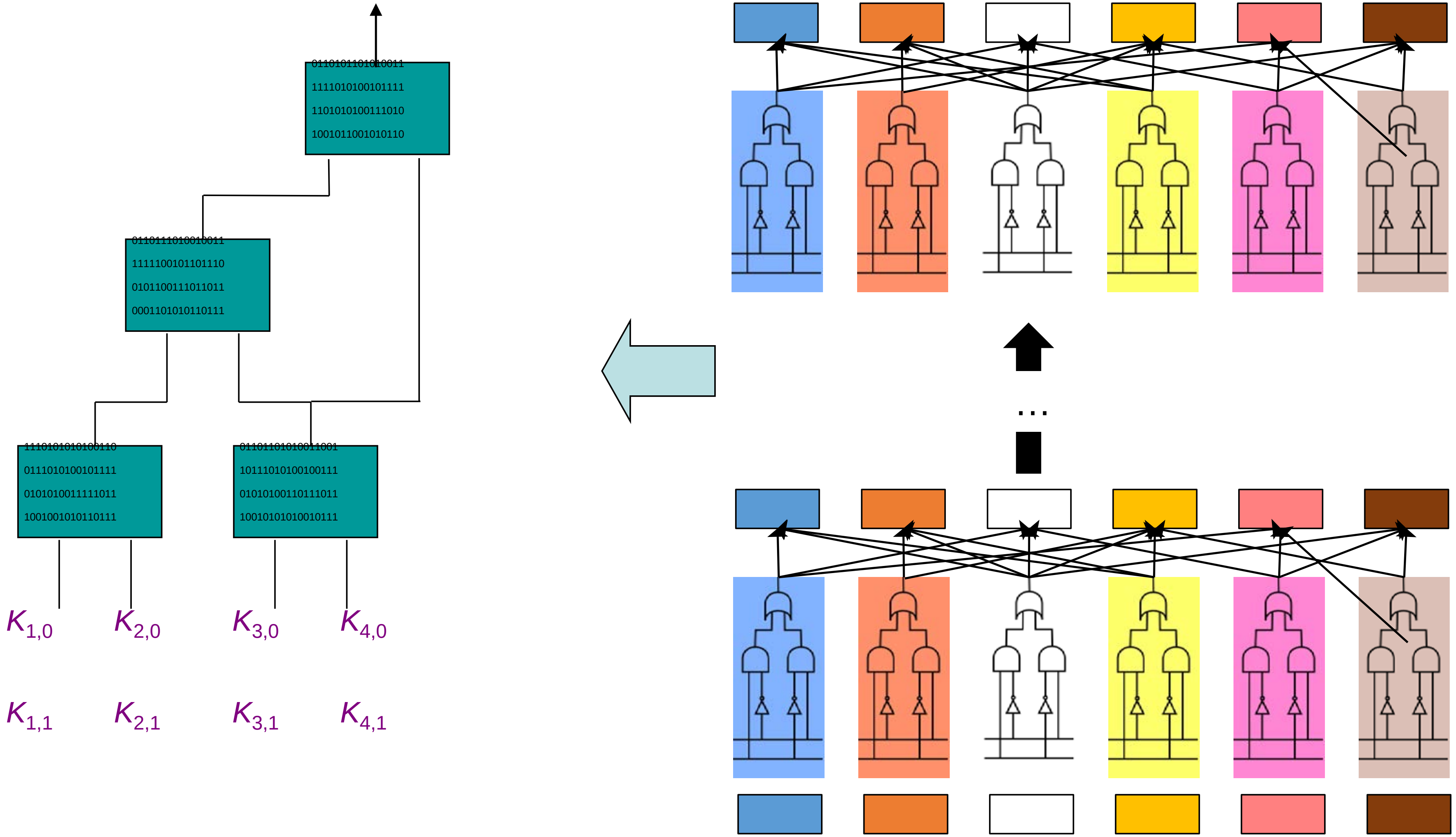
The MPRE is "simple":

Each output y is either:

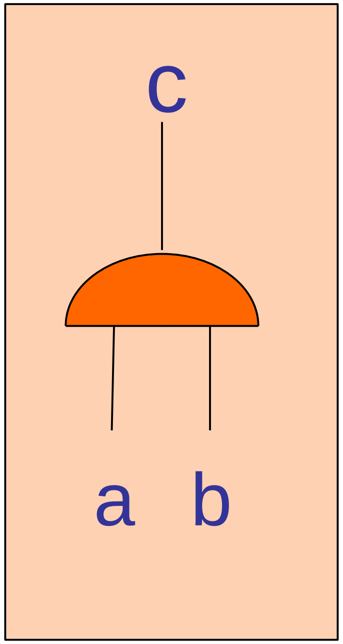
- output of local computation
- Value sent by another party



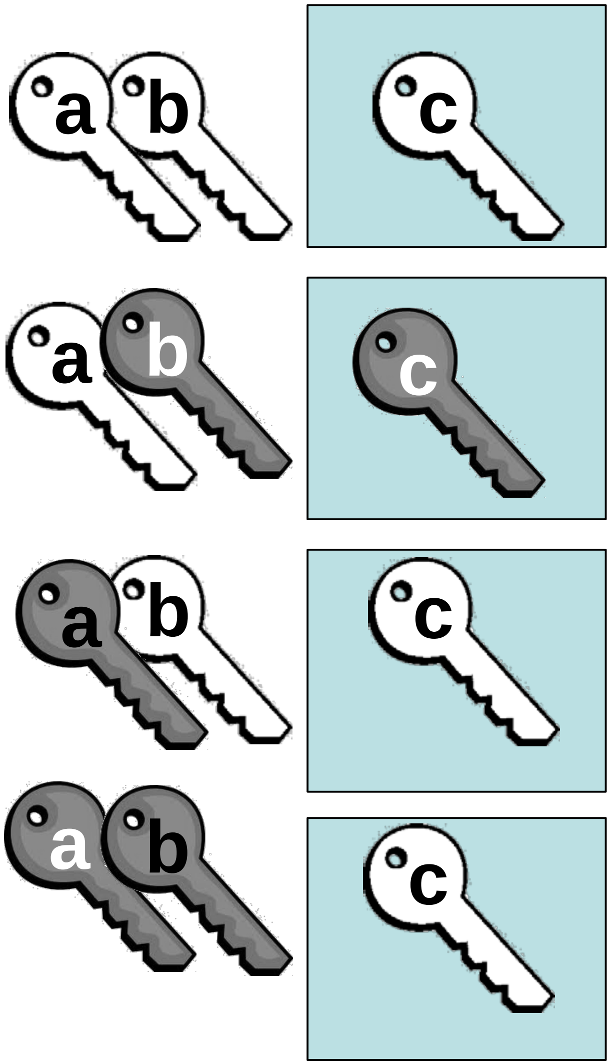
Step 2: re-encode via perfect Garbled Circuit



Case 1: Local Computation Gates



Orange party



Randomness per wire:

- mask bit

owned by orange party

- 2 keys

shared between all

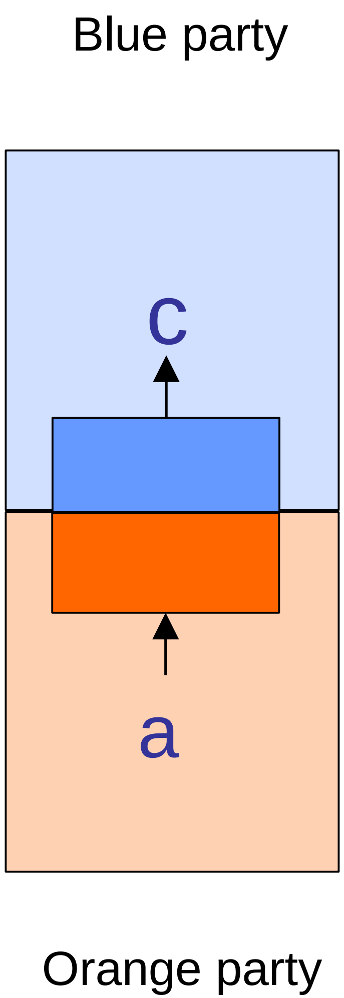
Release:
4 ciphertexts per gate
For all parties

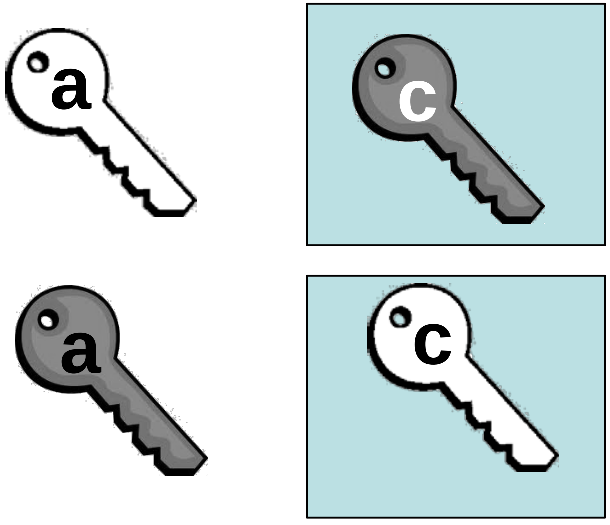
Degree-2
(after preprocess)!

- since masks of a,b,c

are known to same party

Case 2: Transmission Gates





Randomness per wire:

- mask bit

owned by orange/blue

- 2 keys

shared between all

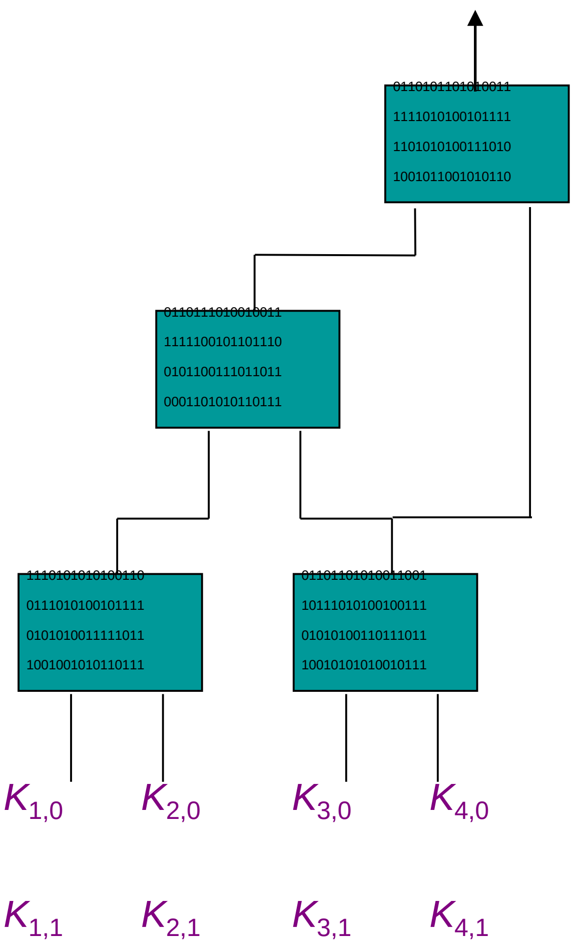
Release for all:
2 ciphertexts per gate

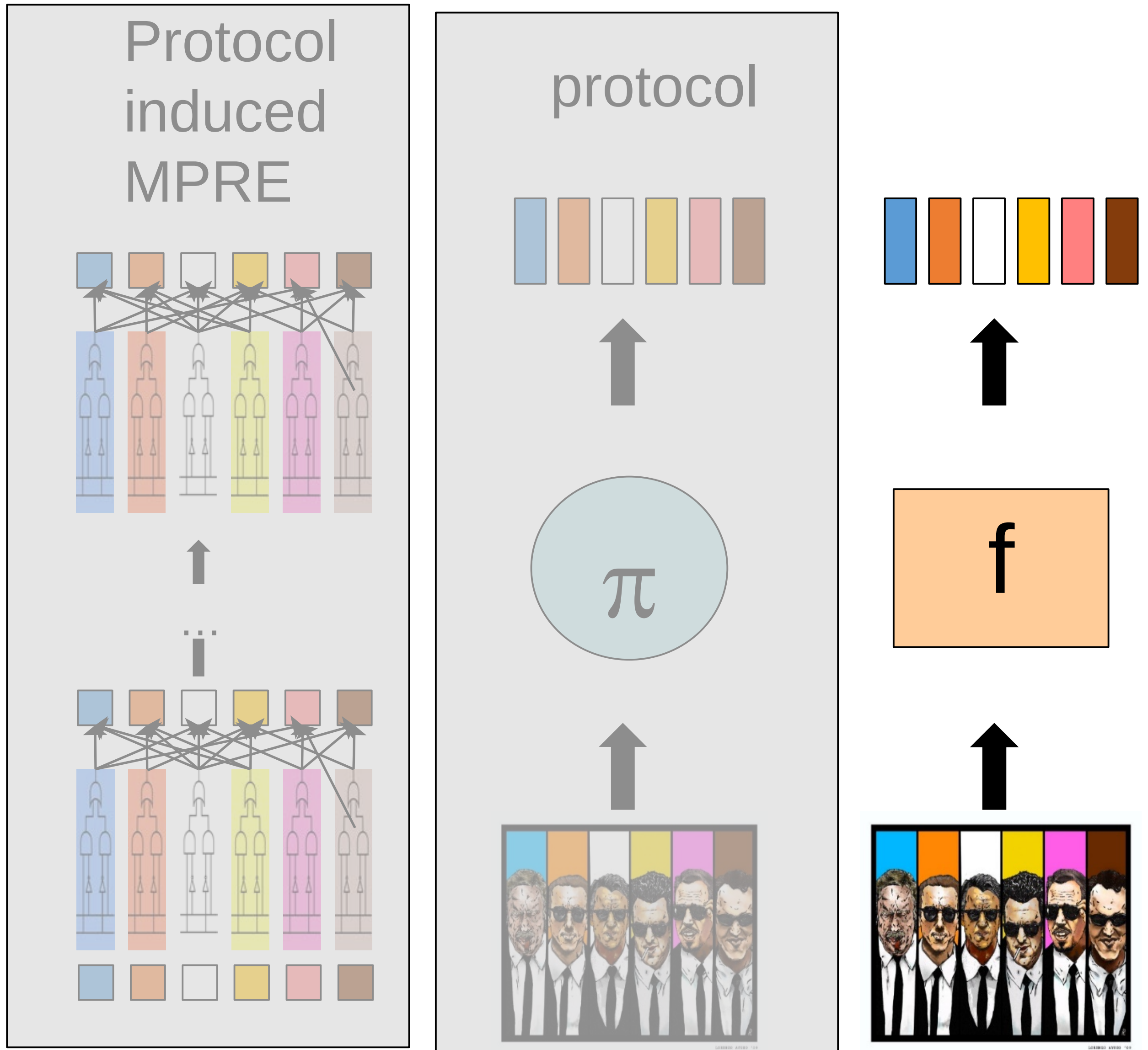
Degree-2 !

- since deg(gate)=1

Putting it all together

GC-based MPRE
Effective deg-2





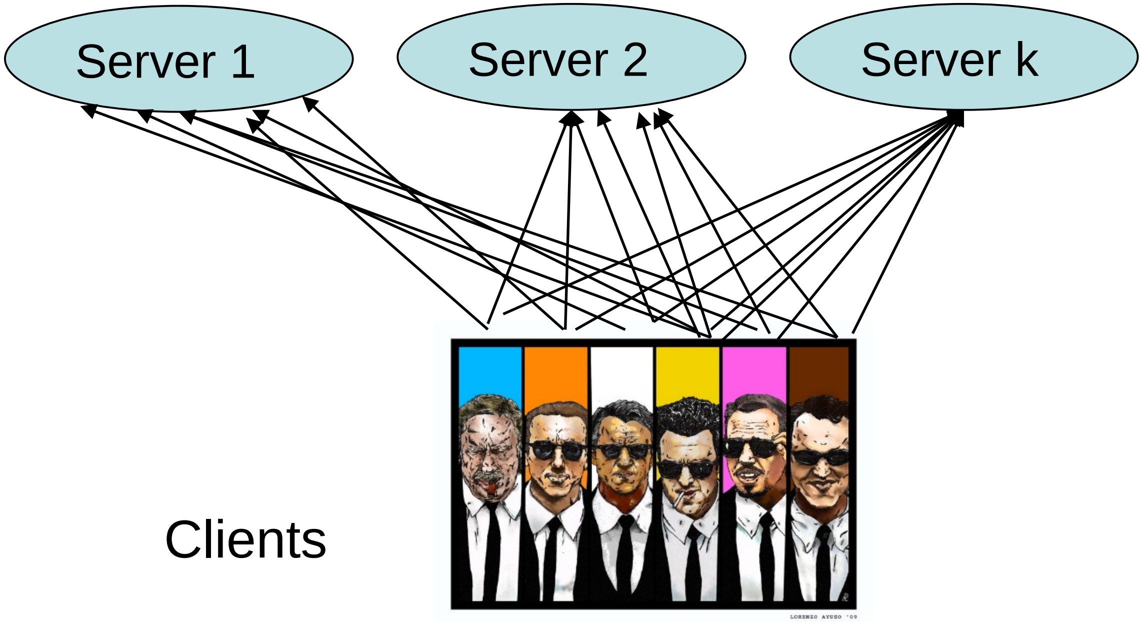
Conclusion

Assuming honest majority and passive adversary:

- Every function has perfect 2-round protocol
  - Efficient for NC1, log-space
  - Computational variant for poly-size circuits using OWFs

Conclusion

- Practical relevance?
  - 2-round protocols easily transfer to client-server model [Ishai-Damgard '05]



Conclusion

- Practical relevance?
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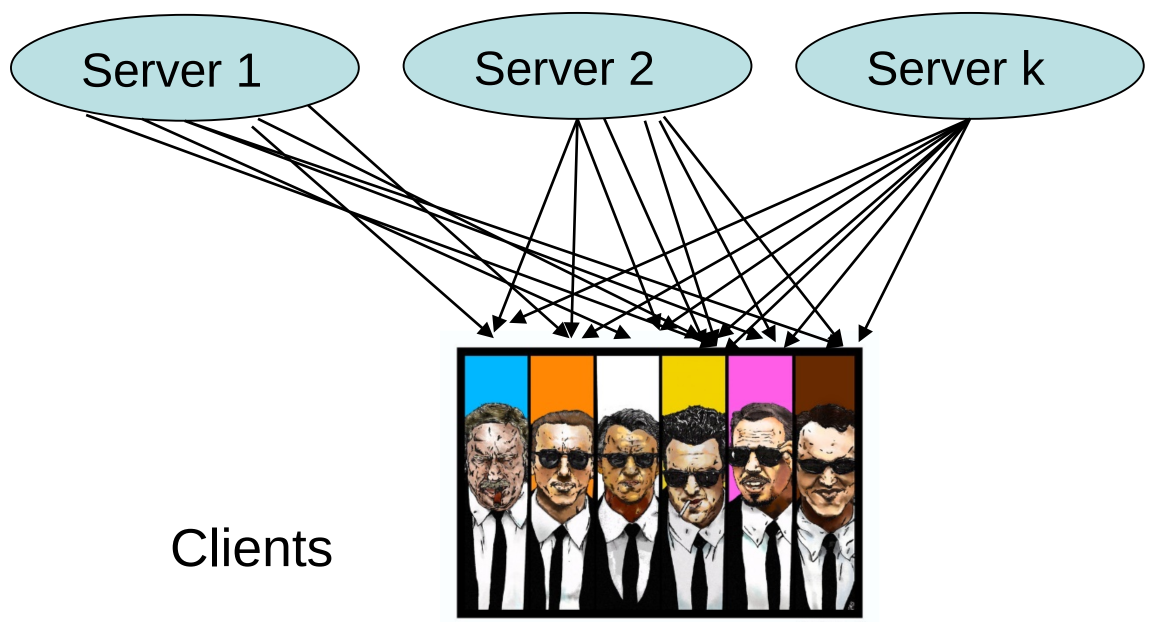
Private as long as majority of the servers & clients are honest



Conclusion

Multiparty Randomized Encoding of Functionalities

- Useful concept

- Other applications?

Thank You