Active inference with function learning for robot body perception

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***Index Terms*—Active inference, body estimation and learning, predictive coding, bio-inspired perception**

***Abstract*—Any variation on the robot characteristics or the interaction medium makes the robot difficult to adapt using model based standard methods. This paper analyses the suitability of the active inference framework for continual learning in artificial agents for providing adaptive body perception and control. The approach presented is inspired on one of the most influential theories about the Bayesian brain: the predictive processing theory modelled as dynamic Bayesian estimation. The underlying idea is to infer the most plausible body state by means of the predictive error: the difference of the expected sensory information, produced by a generative model, and the observed sensations. We theoretically address the advantages and disadvantages of this mathematical model and test it in simulation on a robot with a 2DOF robotic arm and a mounted monocular camera.**

I. Introduction

Predictive processing, or its biologically counterpart predictive coding, is a well know mathematical model under the Bayesian assumption, introduced by [1], that tries to explain how the brain perceives the world. Under this paradigm, perception is a minimization of the prediction error between the inner model, modulated by higher cortical layers, and the observed variables (sensors) [1], [2]. Accordingly, active inference [3] is the extension of this model when formalizing perception as a bound relation between sensors and actions. Here, the action also plays the role of minimizing the error prediction. Intuitively, we could either minimize the error by acting on the world or changing our beliefs [3]. The actions are computed as a result of an inference process closely related to the concept of planning as inference described by [4].

Although active inference provides clear advantages over other methods, the majority of the works in the literature assume that the generative models of the environment are known a priori or tuned to obtain an specific behaviour [3], [5]. This highly contradict the enactivist approach, where the agent progressively learns its body by interaction [6] and uses body perception to understand the world [7]. Moreover, just a few works have tried to bring this computational model to robotics. For instance, in [8] they tested the model on a simulated PR2 robot. Another interesting approach was taken in [5] where they modelled a sort of Braitenberg vehicles [9] by means of active inference. Methods that exploit learning use recurrent neural networks like the seminal work from Tani and its application for imitation using a simulated icub robot [2]. On the other hand, we recently proposed in [10] a similar model for robotic body multimodal estimation using Gaussian processes for learning the generative process. We successfully applied the computational model to replicate the passive rubber hand illusion on a robot [11]. However, we only addressed static perception.

This paper presents body estimation and control as a free-model active inference problem combining state-of-the-art regressors [12] with free energy lower bound minimization [13]. The major advantages of this approach is that only forward models need to be learnt, and each sensing modality can be computed separately. However, in a long life learning scheme, we show that obtaining the generative functions for the observation model and the state estimation is already a hard task. The paper is structured as follows: Sec. II explains the active inference model for body estimation; sec III describes the robot model and discuss the results; and finally Sec. IV summarizes the work.

II. Active inference model for robot body perception

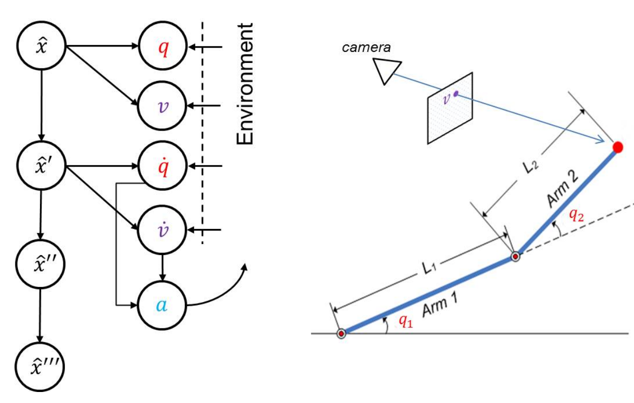

Fig. 1. Model description. Hidden (black) and observable variables (red and purple) and their relation with the environment.

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We cast body perception as a Bayesian inference problem where the body configuration x is inferred using the sensory information by applying Bayes rule:

$$p(x|s) = \frac{p(s|x)p(x)}{p(s)} \quad (1)$$

where $p(s|x)$ is the sensory consequence of being in state x and $p(x)$ is the prior belief of the internal variables. The marginalization of the likelihood over all the possible body states could be intractable. However, we can apply variational free energy approximation [1], [10], [13]. The core idea is to minimize Kullback-Leibler divergence between a distribution $q(x)$ that is encoded in the brain dynamics and the true posterior $p(x|s)$.

$$\begin{aligned} D_{KL}(q(x)||p(x|s)) &= \int q(x) \ln \frac{q(x)}{p(x|s)} dx \\ &= \int q(x) \ln \frac{q(x)}{p(x,s)} dx + \ln p(s) \\ &= \int q(x) \left[\ln q(x) - \ln p(x,s)\right] dx + \ln p(s) \\ &= F + \ln p(s) \end{aligned} \quad (2)$$

Note that minimizing F we reduce $D_{KL}(q(x)||p(x|s))$ as $\ln p(s)$ does not depend on $q(x)$.

Instead of approximating $p(x|s)$ with the whole $q(x)$ distribution, the agent model is a delta distribution $\delta(x - \hat{x})$ that makes $\hat{x}$ the mean of the approximating density [14]. Thus, we can remove the integrals simplifying the free energy to:

$$F = -\ln p(\hat{x}, s) = -(\ln p(s|\hat{x}) + \ln p(\hat{x})) \quad (3)$$

Generalizing for a dynamical system of order n $\{x_1, \ldots, x_n\} \equiv \{x, x', x'', \cdots\}$ with m sensors, the joint probability becomes:

$$F = -\sum_{i=0}^{n}\sum_{j=0}^{m} \ln p(s_j|\hat{x}_i) + \sum_{i=0}^{n} \ln p(\hat{x}_{i+1}|\hat{x}_i) \quad (4)$$

A. Free Energy Framework for Body Perception

Without loss of generality, we define the robot, depicted in Figure 1, as a 2DOF arm with a camera mounted able to see the end-effector. The input sensors are the position of the end-effector in the visual field v and the joint angles q as well as their velocities $\dot{v}$ and $\dot{q}$. The latent variables $\hat{x}$ represent the estimation of the body. We have assumed, conversely to the general predictive processing framework that the brain states are able to encode *position* and *velocity* of body configuration defined as the joint angles.

Hence, we further define $\mu = \{\mu, \mu', \mu'', \mu'''\}$ as the brain variable that represents $\hat{x}$ up to 3rd order dynamics. Assuming the independence of these variables, we express the likelihood of sensing s given an estimated body state μ and the prior for the current model as:

$$P(s|\mu) = p(q|\mu)p(\dot{q}|\mu')p(v|\mu)p(\dot{v}|\mu') \quad (5)$$

$$P(\mu) = p(\mu'|\mu)p(\mu''|\mu')p(\mu'''|\mu'') \quad (6)$$

Note that we have replaced s with the actual sensors variables q and v. When substituting into Eq. 4 we observe that the free energy is the summation of the likelihoods of getting a sensor value given our belief of body configuration (e.g., location of the end-effector in the visual field given the joint angles $p(v|\mu)$), plus the summation of the prior belief $p(\mu)$.

1) Generative functions: According to the predictive processing framework [3] the brain is able to learn approximated generative functions of the world dynamics given the brain state. For body perception we define the function $g(\mu)$, which predicts the sensor value given the current belief of the body configuration/state, and $f(\mu)$, which predicts the dynamics of the system. This predictors of forward models must be continuously learnt to enable perception - see Sect. II-B.

Hence, we shall define the sensor likelihood $p(q|\mu)$ as a Normal distribution with mean $g_q(\mu)$ and variance σ_q [14]:

$$p(q|\mu) = \frac{1}{Z} \exp\left[-(q - g_q(\mu))^2/2\sigma_q\right] \quad (7)$$

$$p(\dot{q}|\mu') = \frac{1}{Z'} \exp\left[-(\dot{q} - \frac{\partial g_q(\mu)}{\partial \mu}\mu')^2/2\sigma_{\dot{q}}\right] \quad (8)$$

where $Z = \sqrt{2\pi\sigma_q}$.

Analogously, body dynamics $p(\mu'|\mu)$ follows a Normal distribution with mean $f(\mu)$ and variance σ_μ:

$$p(\mu'|\mu) = \frac{1}{W} \exp\left[-(\mu' - f(\mu))^2/2\sigma_\mu\right] \quad (9)$$

$$p(\mu''|\mu') = \frac{1}{W'} \exp\left[-(\mu'' - \frac{\partial f(\mu)}{\partial \mu}\mu')^2/2\sigma_{\mu'}\right] \quad (10)$$

where $W = \sqrt{2\pi\sigma_\mu}$. Note that we have removed $p(\mu'''|\mu'')$ assuming that it is just noise [13].

2) Body perception as free energy optimization: Calculating the free energy by substituting in Eq. 4 with the distribution functions defined in Eq. 5 and 6 we obtain:

$$\begin{aligned} F = &\ln 1/W + \ln 1/W' + \ln 1/Z + \ln 1/Z' \\ &- \frac{1}{2\sigma_\mu}(\mu' - f(\mu))^2 - \frac{1}{2\sigma'_\mu}(\mu'' - \frac{\partial f(\mu)}{\partial \mu}\mu')^2 \\ &- \frac{1}{2\sigma_q}(q - g_q(\mu))^2 - \frac{1}{2\sigma_{\dot{q}}}(\dot{q} - \frac{\partial g_q(\mu)}{\partial \mu}\mu')^2 \\ &- \frac{1}{2\sigma_v}(v - g_v(\mu))^2 - \frac{1}{2\sigma_{\dot{v}}}(\dot{v} - \frac{\partial g_v(\mu)}{\partial \mu}\mu')^2 \end{aligned} \quad (11)$$

Body perception is then reduced to inferring μ by minimizing F through gradient descent [3], [10]:

$$\begin{aligned} \frac{\partial F}{\partial \mu} = &\frac{1}{\sigma_\mu}(\mu' - f(\mu))\frac{\partial f(\mu)}{\partial \mu} + \frac{1}{\sigma'_\mu}(\mu'' - \frac{\partial f(\mu)}{\partial \mu}\mu') \cdot \partial\partial f_\mu \\ &+ \frac{1}{\sigma_q}(q - g_q(\mu))\frac{\partial g_q(\mu)}{\partial \mu} + \frac{1}{\sigma_{\dot{q}}}(\dot{q} - \frac{\partial g_q(\mu)}{\partial \mu}\mu') \cdot \partial\partial g_{q\mu} \\ &+ \frac{1}{\sigma_v}(v - g_v(\mu))\frac{\partial g_v(\mu)}{\partial \mu} + \frac{1}{\sigma_{\dot{v}}}(\dot{v} - \frac{\partial g_v(\mu)}{\partial \mu}\mu') \cdot \partial\partial g_{v\mu} \end{aligned} \quad (12)$$

3) Model simplifications and functions meaning: Without loss of generality we enforce some assumptions to simplify the computation of the free energy based on the problem formulation described in Fig. 1. We first set body configuration latent variable as the joints angles q. Then, in the case of having a sensor that provides that information, μ becomes the predictor of the body state: $g_q(\mu) = \mu$[1]. We apply the same assumption for $\dot{q}$ by estimating the joint angle velocity: $\frac{\partial g_q(\mu)}{\partial \mu}\mu' = \mu'$. Secondly, we define $f(\mu)$ as the change on the body configuration or "velocity". Finally, for the sake of clarity we rename $\partial f(\mu)/\partial \mu$ as $h(a, \mu, \mu')$, a function that computes the acceleration of the body joint angles depending on the force/torque applied the current state and the "velocity"[2].

Under the previous assumptions and definitions Eq. 12 becomes:

$$\begin{aligned}\frac{\partial F}{\partial \mu} &= \frac{1}{\sigma_q}(q - \mu) \\ &+ \frac{1}{\sigma_v}(v - g_v(\mu))\frac{\partial g_v(\mu)}{\partial \mu} \\ &+ \frac{1}{\sigma_\mu}(\mu' - f(\mu))\frac{\partial f(\mu)}{\partial \mu} \\ &+ \frac{1}{\sigma'_\mu}(\mu'' - h(a, \mu, \mu'))\frac{\partial h}{\partial \mu}\end{aligned} \quad (13)$$

Analogously for μ':

$$\begin{aligned}\frac{\partial F}{\partial \mu'} &= \frac{1}{\sigma_{\dot{q}}}(\dot{q} - \mu') + \\ &+ \frac{1}{\sigma_{\dot{v}}}(\dot{v} - g_{\dot{v}}(\mu'))\frac{\partial g_{\dot{v}}(\mu)}{\partial \mu'} \\ &+ \frac{1}{\sigma_\mu}(\mu' - f(\mu))\frac{\partial f(\mu)}{\partial \mu'} \\ &+ \frac{1}{\sigma'_\mu}(\mu'' - h(a, \mu, \mu'))\frac{\partial h}{\partial \mu'}\end{aligned} \quad (14)$$

The last order μ'':

$$\frac{\partial F}{\partial \mu''} = \frac{1}{\sigma'_\mu}(\mu'' - h(a, \mu, \mu')) \quad (15)$$

4) Brain variables dynamics to infer the latent space: In order to compute the brain variables we define their differential equations as [3]:

$$\dot{\mu} = D\mu - \frac{\partial F}{\partial \mu} \quad (16)$$

Specifically for 3rd order dynamics Eq. 16 becomes[3]:

$$\begin{aligned}\dot{\mu} = \mu' + \Bigg[&\frac{1}{\sigma_q}(q - \mu) + \frac{1}{\sigma_v}(v - g_v(\mu))\frac{\partial g_v(\mu)}{\partial \mu} \\ &+ \frac{1}{\sigma'_\mu}(\mu'' - h(a, \mu, \mu'))\frac{\partial h}{\partial \mu}\Bigg]\end{aligned} \quad (17)$$

[1]When the sensors do not provide the angle, e.g., muscle spindle, we cannot use this assumption and the generative function shoul be learnt.

[2]In the original work from Friston a is replaced by a hidden cause or a prior expectation [3].

[3]Note that we have inverted the sign to fulfil Eq. 3 negative free energy.

$$\begin{aligned}\dot{\mu}' = \mu'' + \Bigg[&\frac{1}{\sigma_{\dot{q}}}(\dot{q} - \mu') + \frac{1}{\sigma_{\dot{v}}}(\dot{v} - g_{\dot{v}}(\mu'))\frac{\partial g_{\dot{v}}(\mu)}{\partial \mu'} \\ &+ \frac{1}{\sigma'_\mu}(\mu'' - h(a, \mu, \mu'))\frac{\partial h}{\partial \mu'}\Bigg]\end{aligned} \quad (18)$$

$$\dot{\mu}'' = -\frac{1}{\sigma'_\mu}(\mu'' - h(a, \mu, \mu')) \quad (19)$$

With these three equations and the generative functions $g(.)$ and $h(.)$ the robot is able to infer its body configuration integrating all sensory information.

B. Learning the generative functions

We define the learning as obtaining the predictors of the sensor values given the body latent variables. Figure 2 shows the architecture of the system where the perception inference is given by Eq. 16 and the action inference is computed using Eq. 21. Assuming that we do not know the generative functions or predictors $g(\mu)$ and $f(\mu)$ we need to learn them in a unsupervised or semi-supervised manner. There are several ways to approach this non-linear function learning. In [10] we proposed Gaussian Process (GP) regression for learning the functions as we can easily compute the partial derivative with respect to the state. In this paper we get advantage of a high-dimensional space regressor that permits scalable online learning: locally weighted projection regression (LWPR) [15].

In particular, for the body perception test we learnt $h(a, q, \dot{q})$ and $g_v(q)$ using LWPR and in order to generate sample points we used a matsuoka oscillator as a first stage input force (i.e., torque).

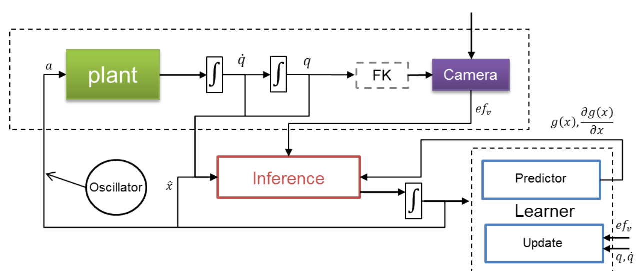


Fig. 2. Overall architecture for the free-model active inference system. The information available is: the joint position sensors and their velocity, and the 2D location of the end-effector on the camera field of view.

C. Active Inference

Under this paradigm, the action plays a core role on the optimization. It also improves the approximation of the real distribution. Thus, reducing the prediction error by minimizing the free energy:

$$a = \arg\min_a F \quad (20)$$

Using the gradient descent formulation the action is computed as:

$$\begin{aligned}\dot{a} &= -\frac{\partial F}{\partial a} = -\frac{\partial s}{\partial a}\frac{\partial F}{\partial s} \\ &= -\left[\frac{\partial q}{\partial a}\frac{(q-\mu)}{\sigma_q} + \frac{\partial v}{\partial a}\frac{(v-g(\mu))}{\sigma_v}\right. \\ &\left.+\frac{\partial \dot{q}}{\partial a}\frac{(\dot{q}-\mu')}{\sigma_{\dot{q}}} + \frac{\partial \dot{v}}{\partial a}\frac{(\dot{v}-\mu')}{\sigma_{\dot{v}}}\right]\end{aligned} \tag{21}$$

III. Results

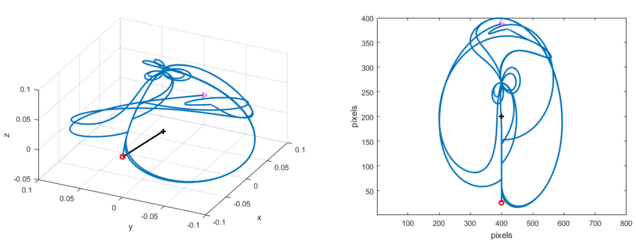
We tested the computational model on a simulated 2DOF robot arm in matlab simulink generated by with the toolbox [16]. The robot, represented in 1, is driven by the following dynamics:

$$\ddot{q} = M\backslash(\tau - C(q)\dot{q} - G(q) - \beta\dot{q}) \tag{22}$$

where τ, M, C, G and β are the torque, inertia, Coriolis, gravitational and friction matrices respectively.

Figure 3 shows an instance of the end-effector 3D and visual trajectory generated at the training stage.

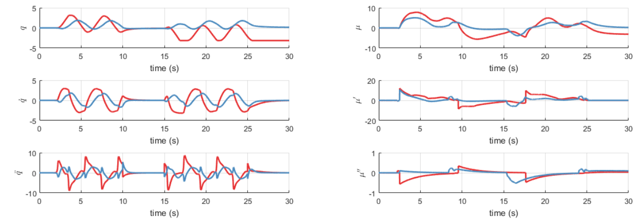
First we test the proposed perception approach without the action. Figure 4 shows how the estimation μ tries to approximate the real values of the robot body joints. However, there is a big mismatch in high order latent variables. The reason is that the inference is highly sensitive to the generative function $h(a, q, \dot{q})$. Fig. 5 shows the predictor depending on a and μ'. There is a considerable difference if we compare with $\ddot{q}$. This means that the regressor did not properly learn the robot dynamics and both the predicted Jacobian and the values are not temporally smooth. The method used in [10] for learning the generative functions gave better results in terms of interpolating the predictions.



(a) 3D trajectory (b) Visual trajectory

Fig. 3. End-effector trajectory generated using a matsuoka oscillator in world coordinates and in the visual frame.

Finally, in order to test the active inference (Eq. 21) we substitute the unreliable predictor of $h(a, \dot{q})$ by its ground truth value $\ddot{q}$ (Fig. 6(c)). We define the goal or desire state of the robot as $\mu = [0.1, -0.1]$ joint angles and zero velocity $\mu' = [0, 0]$. This goal is assumed to be defined by high order layers in the predictive processing scheme. Figure 6(a) shows the robot behaviour and Fig. 6(b) the latent space dynamics. Note that although q and μ are quite similar, the second and third order differ. The robot is able to reach the desired body joint angles position by means of the action through the minimization of the prediction error (Fig. 6(d)) .



(a) Ground truth (b) Body estimation

Fig. 4. Perception of the body joint angles.

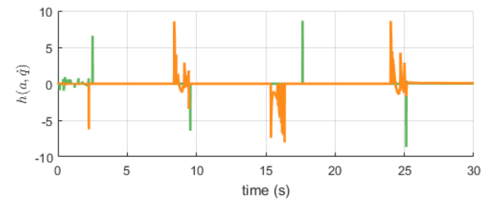
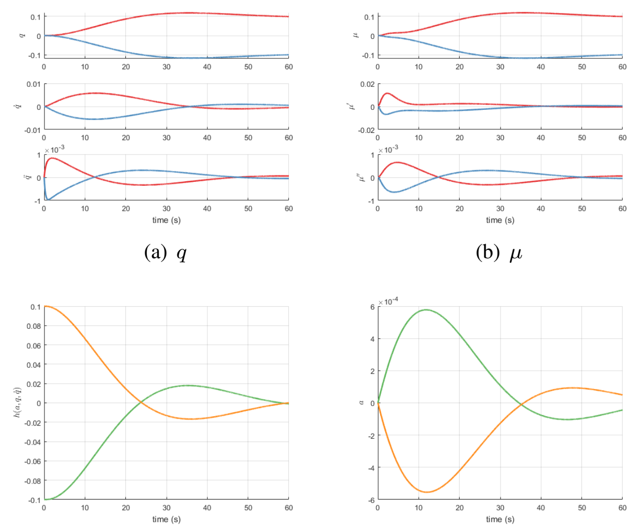


Fig. 5. Acceleration $\ddot{q}$ predictor using LWPR.



(a) q (b) μ

(c) $h(a, \mu, \mu')$ (d) Force action

Fig. 6. Active inference with prior joint angles. Desired state: $\mu = [0.1, -0.1]$ and $\mu' = [0, 0]$. The action moves the robot towards reducing the prediction error.

IV. Conclusion

We have presented body action and perception from the predictive processing framework and analysed the suitability of learning from scratch the needed generative functions during the movement of the robot. Results show how body inference depends on learning the state forward dynamics, yielding to a hard problem. Moreover, the necessity of computing the Jacobian of the observation model and the state dynamics with respect to the latent space does not makes this approach simpler compared with the inverse dynamics approach. However, in theory, the free energy optimization methodology could solve the discrepancies between the learnt generative model and the real world dynamics, as well as improving scalability in multisensory data assimilation.

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