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A Morphological Transform to Increase Geometric Invariance and Generalization in Convolutional Neural Networks

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Convolutional Neural Networks are nowadays one of the most useful paradigms in deep learning and provide significant state-of-the-art results on image classification and image/object segmentation. A suitable mathematical theory of deep convolutional neural networks (DCNN) for feature extraction has been considered previously [2, 7], in particular concerning the robustness and geometric invariance (i.e., sensitivity to image deformation).

Distance function is a classical transform to represent a shape as a function which provides an extended morphological description since its level sets are associated to isotropic erosions and dilations of the shape. Distance function has been used in several DCNN architectures for object segmentation and shape classification [5, 6].

Our goal is to theoretically justify the interest of the distance function to increase the geometric invariance in DCNN for feature extraction from general images in classification tasks. Firstly, we review the morphological and stochastic properties of the Molchanov distance function (MDF) [3], which is the natural extension of the distance transform for gray-scale images. Secondly, we prove our result of Lipschitz stability and deformation sensitivity bound of DCNN descriptors, in the case where these descriptors are learnt from images represented by their MDF. Thirdly, we provide some experiments to illustrate the performance and robustness of classification after image perturbation. Additionally, we consider the behavior of our morphological representation based on the MDF vs. the original image representation against adversarial attacks [1, 4].

References

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