# Abstract

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## *Extensions of the* **M-tortuosity** *for heterogeneity assessment and grayscale images characterization*

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Tortuosity is among the foremost of the topological descriptors. Unfortunately, it has not a simple or universal definition [1–2]. We propose two new topological descriptors based on geometric tortuosity notion [3], more specifically based on the *M-tortuosity* concept [4]. First, by using the *M-tortuosity* formalism to quantify heterogeneity and second, by extending its definition to gray-level images. *M-tortuosity* descriptor already handles disconnections in complex interconnected microstructures without definition of arbitrary entry and exit. Our descriptors are named *H-tortuosity* and *F-tortuosity*, respectively. Both are based on Monte Carlo method, the first one uses dilating spheres to assess an overall geometric tortuosity value for a given distance. The second descriptor uses the functional definition of geodesic distance transform [5–6]. The particularity of the *F-tortuosity* is that the functional geodesic distance $FD_G$ is used as a guide into the grayscale maze, for the computation of the *projected functional geodesic distance* $FD_G^{\perp}$, defined as the length of the shortest path on the grayscale image, orthogonally projected on the hyperplane of intensity equal to zero.

$N$ points $p_n$ are sampled in the image such that $p_n \neq c$, the center of mass of the porous volume. For each pair $(p_n, p_m)$, the *functional geometric tortuosity* $\tau_{f_{n,m}}$ is computed as being the ratio of $FD_G^{\perp}$ and their Euclidean distance $D$. The *F-coefficient* $C_{f_n}$, attached to the starting point $p_n$, is defined as in [4] using the harmonic mean of $\{\tau_{f_{n,m}}\}_{m\in[\![0,N-1]\!],m\neq n}$ weighted by the inverse of their respective geodesic distances. Finally, the *F-scalar* $\tau_F$ is defined, according to [4], as the harmonic mean of $\{C_{f_n}^{-1}\}_{n\in[\![0,N-1]\!]}$ weighted by the inverse of their respective Euclidean distances to $c$. The extension is possible thanks to $FD_G$,

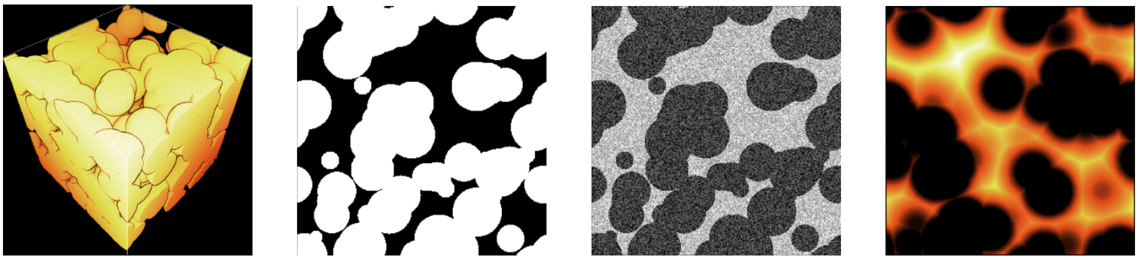
$$FD_G(p_m, p_n; I) = \inf_{\Gamma_f \in \gamma_{f_{p_n,p_m}}} \int_{p_n}^{p_m} \sqrt{1 + (I'(S))^2}\, ds$$

with $\gamma_{f_{p_n,p_m}}$ the set of all paths between $p_n$ and $p_m$ constrained by gray-levels of $I$, $\Gamma_f$ one of these paths and $s$ the arc length. $FD_G$ is used to compute $FD_G^{\perp}$ defined as,

$$FD_G^{\perp}(p_m, p_n; I) = L(\Gamma^*) \qquad \text{with, } \Gamma^* = (\Gamma_f^*)_{\perp}$$

with $\Gamma^*$the orthogonal projection of the shortest path $\Gamma_f^*$ on the hyperplane of intensity equal to zero and $L(\Gamma^*)$ the length of $\Gamma^*$. Such a formulation makes possible the characterization of unsegmented images.

The *F-tortuosity* applied on the distance transform (cf. Fig. 1 (d)) of a binary image, allows to combine tortuosity notion with narrowness, highlighting bottleneck effect. Validation and results of both descriptors, on several multi-scale Boolean schemes [7–10] will be shown (cf, Fig. 1). Their discriminant power will be pointed out. Finally, application on alumina catalyst supports, obtained by electron tomography, will be presented.



(a) (b) (c) (d)

**Figure 1:** (a) 3D visualization of Boolean scheme of spheres (radius = 30, volume fraction = 0.7), (b–c) slice of (a) for reference value computation (b) and validation on noisy image (c), (d) slice of (a) of 3D geodesic distance map from the spheres boundary.

*References*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