Mads Stehr (CSGB)

Asymptotic Variance of Newton-Cotes Quadratures based on Randomized Sampling Points

Joint with Markus Kiderlen

Motivated by the stereological issue of volume estimation from cross-sectional area measurements, we consider the problem of numerical integration when sampling nodes are random, and we suggest to use Newton-Cotes quadrature rules to exploit smoothness properties of the integrand. Compared to the classical Cavalieri estimator based on equidistant sampling, it was shown in previous papers that a Riemann sum approach can cause a severe variance inflation when the sampling points are not equidistant. However, under some integrability conditions on the typical point-distance, we show that Newton-Cotes quadratures based on a stationary point process in $\mathbb{R}$ yield unbiased estimators for the integral and that the aforementioned variance inflation can be avoided if a Newton-Cotes quadrature of sufficiently high order is applied. More specifically, we show that the variance of estimators based on n'th order Newton-Cotes rules decrease at the same rate as the classical Cavalieri estimator (with decreasing point increment), if the integrand is a so-called $(n, 1)$-piecewise smooth function.

By simulations we illustrate the variance improvement of the trapezoidal rule and Simpson's rule for functions with varying smoothness properties.