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Abstract

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Local Minkowski measures for random set geometrical characterization.

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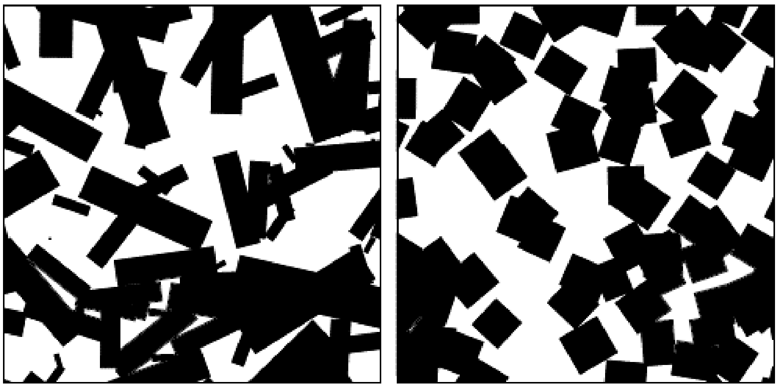

Figure 1: Two Boolean models of rectangles with different parameters, but equal (mean) global Minkowski functionals (global area, perimeter and Euler-Poincaré characteristic).

The physico-chemical behaviour of granular media often rely on the geometrical characteristics of the particles. Quantification of parameters such as granularity, porosity, tortuosity make use of the Minkowski functionals (MFs) of the whole structure. They are also known as intrinsic volumes or quermassintegrals and play an important role in the geometrical characterization of spatial structures (in $\mathbb{R}^2$ and $\mathbb{R}^3$ MFs coincide up to normalization with standard geometrical parameters). Nevertheless, facing the complex spatial structure, the global functionals appear to be not discriminating enough. For example, the simulated images in Figure 1 present the same global area, perimeter and Euler-Poincaré characteristic. Searching for a finer geometrical characterization of the structure, one could consider the local extension of the MFs - the Minkowski measures (MMs). In the present study, these local measures are considered in the Stochastic Geometry framework, i.e. applied to the random closed sets, with the object to determine their behaviour for different typical random spatial structures. The MMs distributions for the simulated germ-grain model are numerically studied.

For a given random closed set Ξ in the extended convex ring of $\mathbb{R}^d$, those are $d+1$ (random) measures on $\mathbb{R}^d$). Particular cases in $\mathbb{R}^2$ are the *area measure* $V_2(\Xi \cap \cdot)$, *perimeter measure* $V_1(\partial\Xi \cap \cdot)$ and Euler-Poincaré measure $V_0(\Xi \cap \cdot)$. In general, for $i < d$ the measures are concentrated on the surface $\partial\Xi$.

The means of local MMs $V_i(\Xi \cap B)$ in the case of statistical homogeneity are given by the product of the d-dimensional volume of B and the constant dependent on Ξ (in $\mathbb{R}^2$ area fraction, perimeter fraction and specific Euler-Poincaré number). Thus, even for a Boolean model of disks, the expectation of local measures giving the first information about Ξ (more precisely, one could derive certain moments of the radius distribution), in general, do not determine the whole distribution. For a bit more complex structure, for example, a Boolean model with rectangular grains as in Figure 1, the individual particle geometry could not be accessed using the global MFs, neither by first moments of local MMs. In order to complete the geometrical characterization, the distributions of MMs $V_i(\Xi \cap B)$ for the Boolean model Ξ are analysed for a special case for the random set B.

A random point uniformly distributed in Ξ determine a "center" of a closed set B (e.g. a center if B is a disk or a centroid of an arbitrary set). Thus only the spatial position of B changes. In Figure 2 for every realisation of a random point uniformly distributed in Ξ, the perimeter of intersection of the boundary $\partial\Xi$ and a disk of radius r centred on x is computed. The correlation between the perimeter measure and particle elongation is evidenced by comparing the distributions for the Boolean models of rectangles with identical characteristics except for elongation ratio.

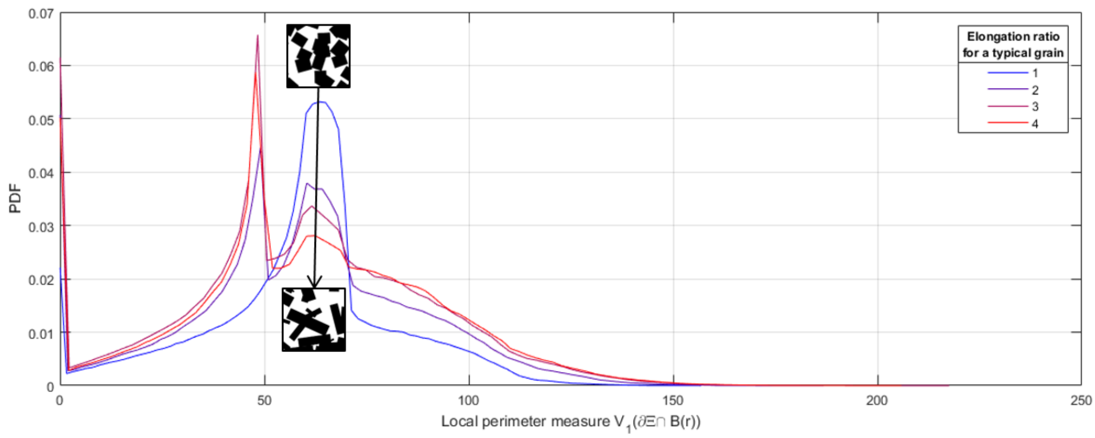

Figure 2: Probability density functions for the local perimeter measure of four Boolean models with rectangular grains of growing elongation ratio. B could fully lie in Ξ which leads to a peak at!0. The peak at $2 \cdot r$ corresponds to the ball centres over the boundary $\partial\Xi$. Here $r = 25$, mean grain length varies in $\{50, 33, 25, 20\}$.

Next steps are the identification (if any) of the relationship of MMs distributions with the quantitative descriptors used in material science (porosity, rugosity, tortuosity, etc.). In the future, the local MMs would be studied for more complex stochastic germ-grain models than the Boolean model.