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Abstract

Quantum Information

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Revisiting the simulation of quantum Turing machines by quantum circuits through the study of causal unitary evolutions.

Joint with John Watrous

Yao [1] proved that $t \geq n$ steps of a quantum Turing machine running on an input of length n can be simulated by a uniformly generated family of quantum circuits with size quadratic in t. We revisit the simulation of quantum Turing machines with uniformly generated quantum circuits, and present a new variant of the simulation method employed by Yao, together with an analysis of it. This analysis reveals that the simulation of quantum Turing machines can be performed by quantum circuits having depth linear in t, rather than quadratic depth, and can be extended to variants of quantum Turing machines, such as ones having multi-dimensional tapes.

Our analysis is based on extending a method of Arrighi, Nesme, and Werner [2] that allows for the localization of causal unitary evolutions. Our extension proves that such localization can still be performed when the causality assumptions are weakened. More specifically, it considers unitary evolutions that do not necessarily act causally on all elements of the space $\mathcal{H}$ they act upon. Instead, the unitary evolutions we consider are only causal when restricted to a subspace that satisfies certain constraints in its relation to the local spaces that together (i.e. through their tensor product) constitute the space $\mathcal{H}$.

References

[1] A. C. Yao, Quantum circuit complexity. *Proc. 34th Annual Symposium on Foundations of Computer Science* (1993): 352-361.

[2] Arrighi, Pablo, Vincent Nesme, and Reinhard Werner. Unitarity plus causality implies localizability. *Journal of Computer and System Sciences* 77.2 (2011): 372-378.