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Abstract

Spectral Theory

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Sharp spectral transition for eigenvalues embedded into the spectral bands of perturbed periodic operators

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In this paper, we consider the Schrödinger equation,

$$Hu = -u'' + (V(x) + V_0(x))u = Eu,$$

where $V_0(x)$ is 1-periodic and $V(x)$ is a decaying perturbation. By Floquet theory, the spectrum of $H_0 = -\nabla^2 + V_0$ is purely absolutely continuous and consists of a union of closed intervals (often referred to as spectral bands). Given any finite set of points $\{E_j\}_{j=1}^N$ in any spectral band of H_0 obeying a mild non-resonance condition, we construct smooth functions $V(x) = \frac{O(1)}{1+|x|}$ such that $H = H_0 + V$ has eigenvalues $\{E_j\}_{j=1}^N$. Given any countable set of points $\{E_j\}$ in any spectral band of H_0 obeying the same non-resonance condition, and any function $h(x) > 0$ going to infinity arbitrarily slowly, we construct smooth functions $|V(x)| \leq \frac{h(x)}{1+|x|}$ such that $H = H_0 + V$ has eigenvalues $\{E_j\}$. On the other hand, we show that there is no eigenvalue of $H = H_0 + V$ embedded in the spectral bands if $V(x) = \frac{o(1)}{1+|x|}$ as x goes to infinity. We prove also an analogous result for Jacobi operators.