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Abstract

Random Systems

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Extremal eigenvalues of critical Erdos-Renyi graphs

Joint with Johannes Alt and Antti Knowles

We complete the analysis of the extremal eigenvalues of the the adjacency matrix A of the Erdős-Rényi graph $G(N, d/N)$ in the critical regime $d \asymp \log N$ of the transition previously uncovered, where the regimes $d \gg \log N$ and $d \ll \log N$ were studied. We establish a one-to-one correspondence between vertices of degree at least $2d$ and nontrivial (excluding the trivial top eigenvalue) eigenvalues of $A/\sqrt{d}$ outside of the asymptotic bulk $[-2, 2]$. This correspondence implies that the transition characterized by the appearance of the eigenvalues outside of the asymptotic bulk takes place at the critical value $d = d_* = \frac{1}{\log 4 - 1} \log N$. For $d < d_*$ we obtain rigidity bounds on the locations of all eigenvalues outside the interval $[-2, 2]$, and for $d > d_*$ we show that no such eigenvalues exist. All of our estimates are quantitative with polynomial error probabilities.

Our proof is based on a tridiagonal representation of the adjacency matrix and on a detailed analysis of the geometry of the neighbourhood of the large degree vertices. An important ingredient in our estimates is a matrix inequality obtained via the associated nonbacktracking matrix and an Ihara-Bass formula. Our argument also applies to sparse Wigner matrices, defined as the Hadamard product of A and a Wigner matrix, in which case the role of the degrees is replaced by the squares of the ℓ^2-norms of the rows.