QMath14: Mathematical Results in Quantum Physics
Aarhus University, 12–16 August 2019

Abstract

Plenary

Torben Krüger (Universität Bonn)

Universal Singularities of Random Matrices

Joint with J. Alt, G. Cipolloni, L. Erdős and D. Schröder

As the dimension of a self-adjoint random matrix tends to infinity, its eigenvalue distribution is well approximated by a deterministic density function. For a broad class of random matrix models with decaying correlations among the entries, determining this function requires solving the *Dyson equation*. Via a detailed analysis of this non-linear matrix equation in asymptotically infinite dimensions down to the scale of the eigenvalue spacing we expose a universal behavior of the eigenvalue density for the underlying random matrix model. The spectrum separates into distinct *bands*. The density of states is positive and analytic inside the bands and exhibits square root growth at the edges. Whenever the gap between two bands closes a cusp singularity forms, showing cubic root growth on either side. Due to a strong *topological rigidity* the number of eigenvalues within each band does not fluctuate and no eigenvalues can be found inside the gaps between the bands with very high probability. Furthermore, the local eigenvalue statistics is universal at all singularities, i.e. independently of the distribution of the entries each edge eigenvalue follows the Tracy-Widom distribution, while at all cusp points the eigenvalues locally form a Pearcey process.