Train-induced vibrations in soil and structures using a mixed-frame-of-reference approach

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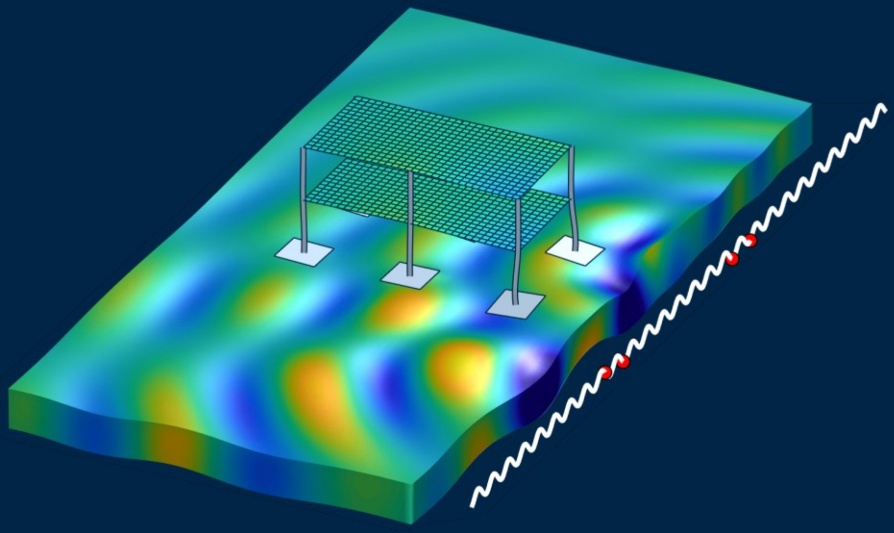
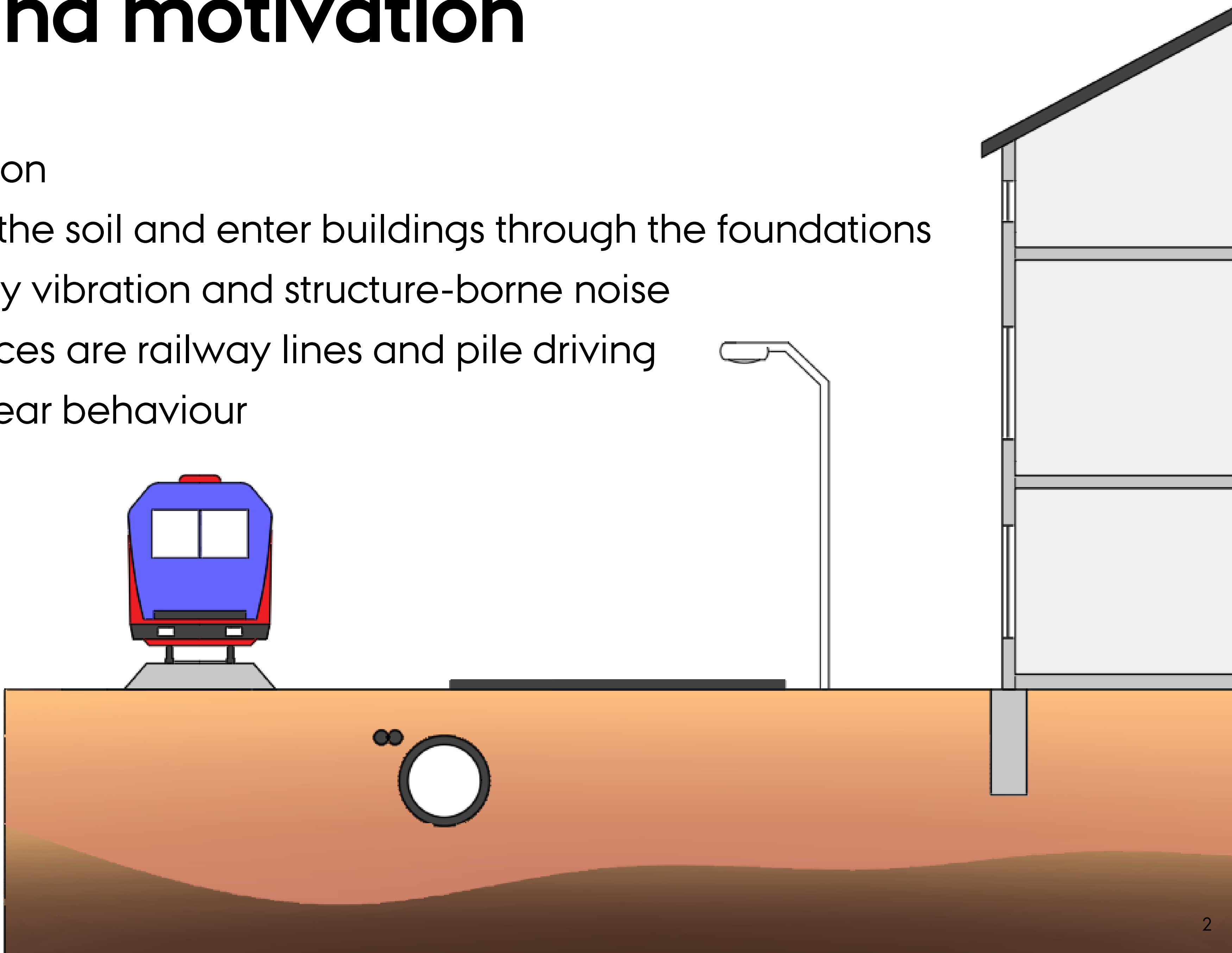
Wood Thilsted Partners



Introduction and motivation

Environmental vibration

- Propagate through the soil and enter buildings through the foundations
- Leads to whole body vibration and structure-borne noise
- Most prevalent sources are railway lines and pile driving
- Low amplitude – linear behaviour



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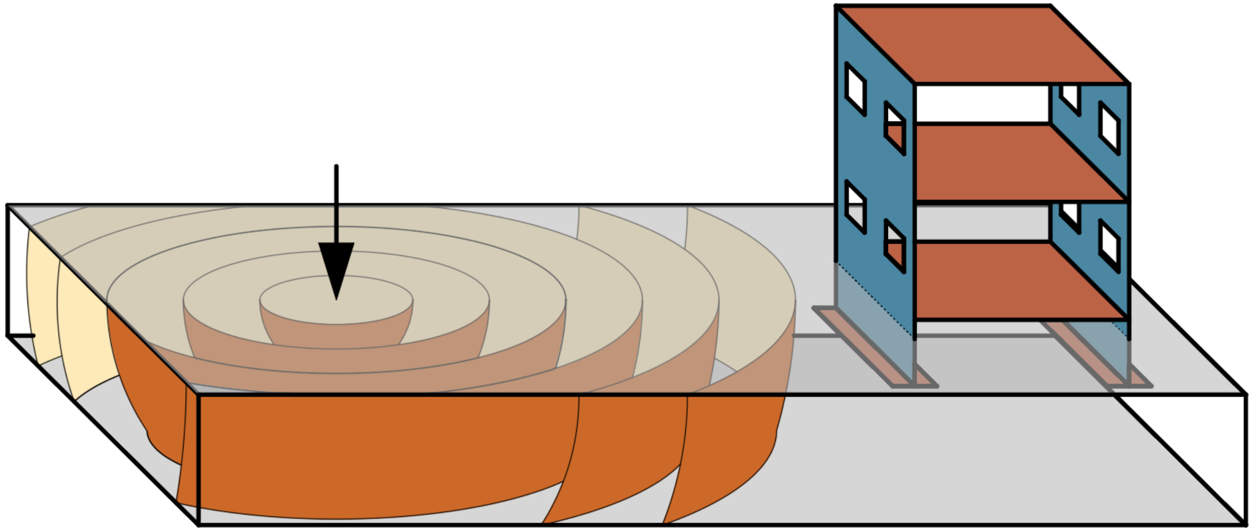
Introduction and motivation

Evaluating environmental vibration

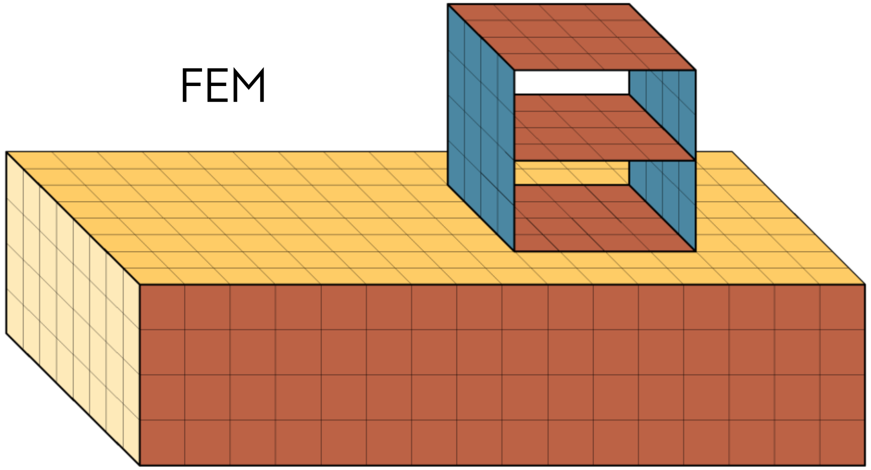
- Dynamic behaviour of soil is a complex phenomena
- In-situ measurement is the most precise approach, but not always feasible
- Empirical models are fast but not precise
- Various numerical methods offer flexibility and precession

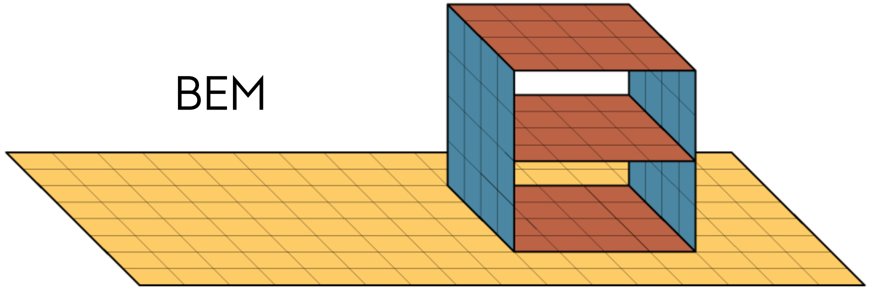


Modelling techniques

Modelling techniques:

- Analytical and semi-analytical methods
- Numerical methods
  - Finite difference method (FDM)
  - Finite element method (FEM)
  - Boundary element method (BEM)
- Various combined methodologies



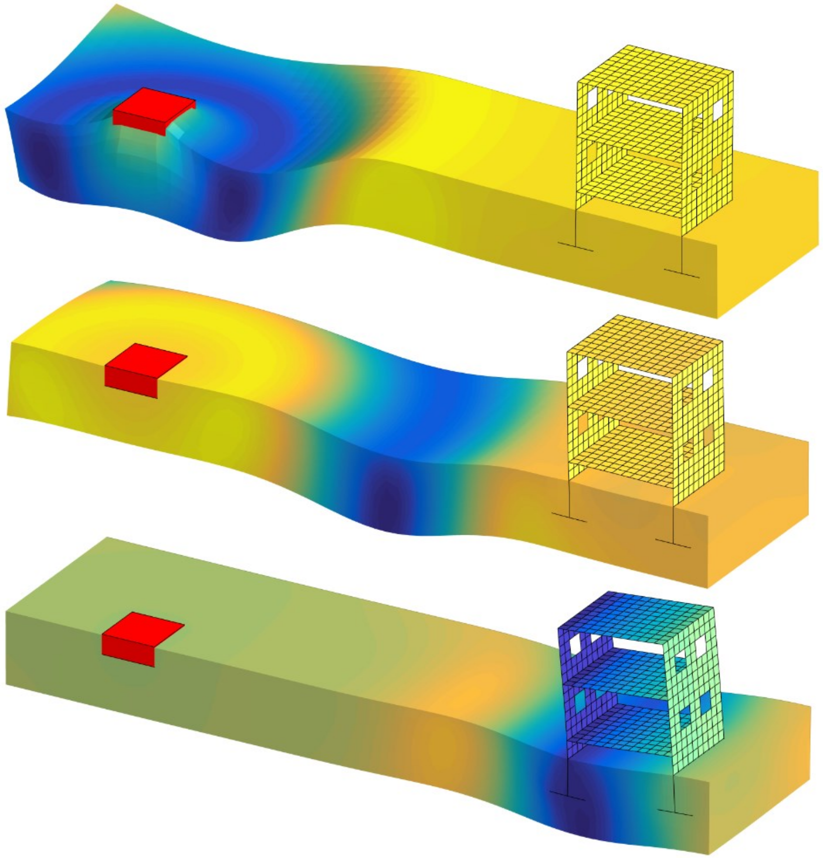


Final choice depends on available resources and needed precision

Semi-analytical soil modelling

Semi-analytical modelling method

- Advantages
  - Quick analytical formulation
  - Solution procedure easy to parallelize
  - Possibility to combine with FEM
- Disadvantages
  - Difficult to model complex cases
  - Complex formulation
  - Only linear / frequency domain behaviour



Semi-analytical soil modelling

- Solution based on the Green's function

$$u_i(x,y,z,t) = \int_{-\infty}^{t} \int_{-\infty}^{0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{ij}(x-x',y-y',z,z',t-t')\, p_j(x',y',z',t')\, \mathrm{d}x'\, \mathrm{d}y'\, \mathrm{d}z'\, \mathrm{d}t'$$

- System is transformed into frequency–wavenumber domain

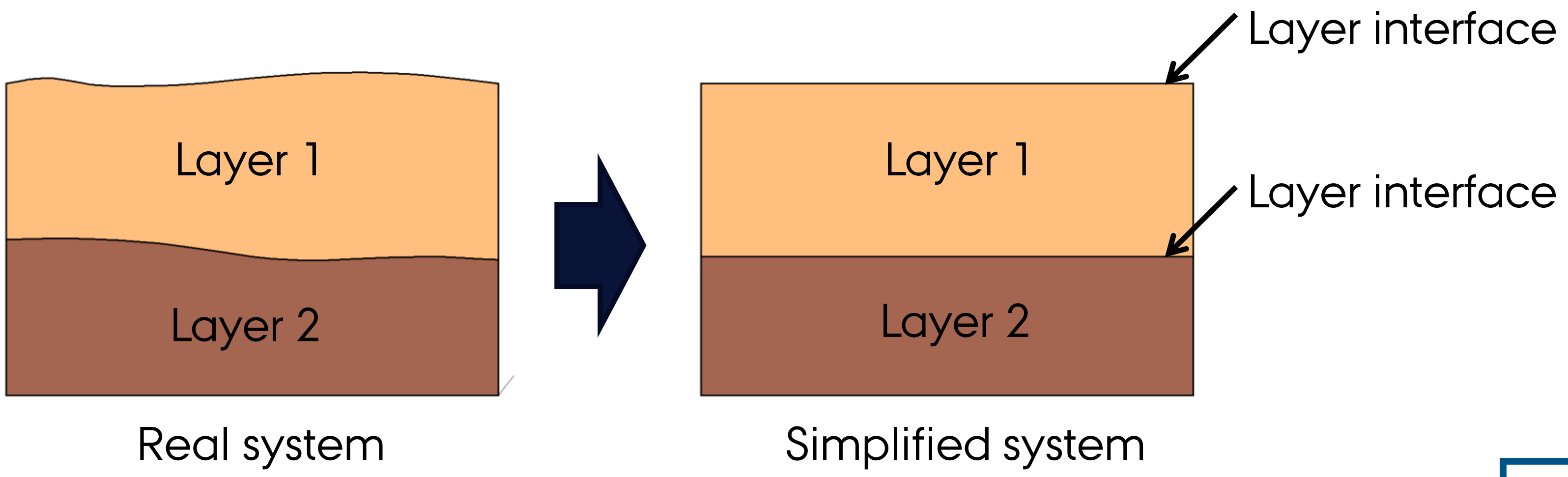
$$\bar{U}_i(k_x,k_y,z,\omega) = \sum_{n=1}^{N_z} \bar{G}_{ij}(k_x,k_y,z,z_n,\omega)\, \bar{P}_{n,j}(k_x,k_y,\omega)$$

  - Analytical expression can be obtained
  - Spatial domain using discrete inverse Fourier transform

Semi-analytical soil modelling

Assumptions:

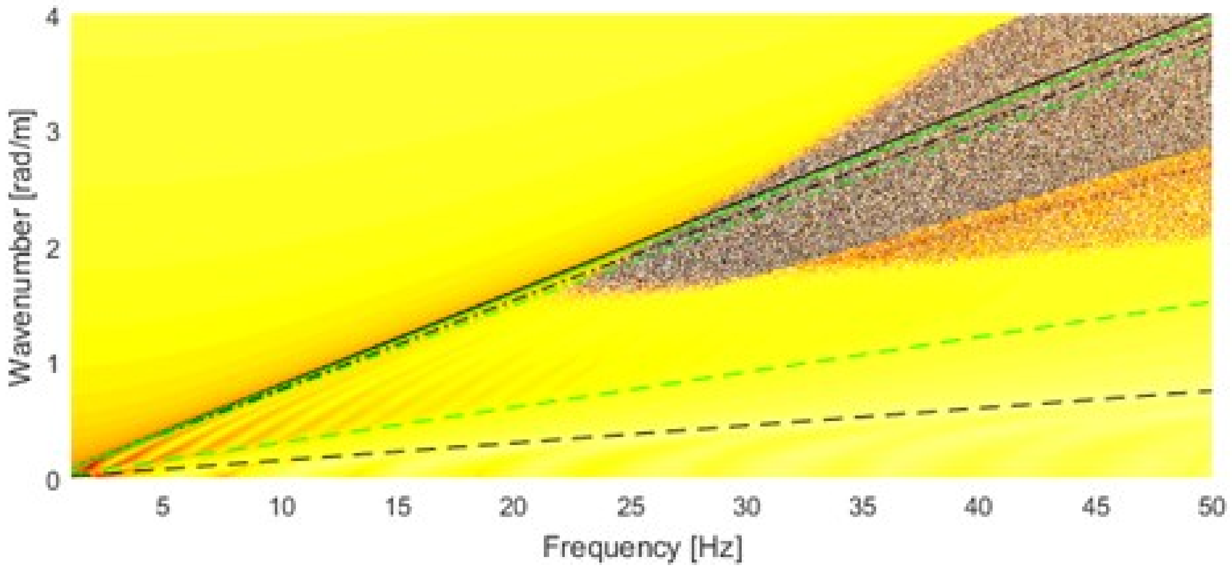
- Soil is linear-viscoelastic with homogeneous layers
- Soil surface and interfaces between layers are perfectly horizontal
- Only frequency domain solution
- Interactions possible only through soil layer interfaces
- Non-reflecting boundaries already part of the Green's function



Semi-analytical soil modelling

Analytical solution to the Green's function

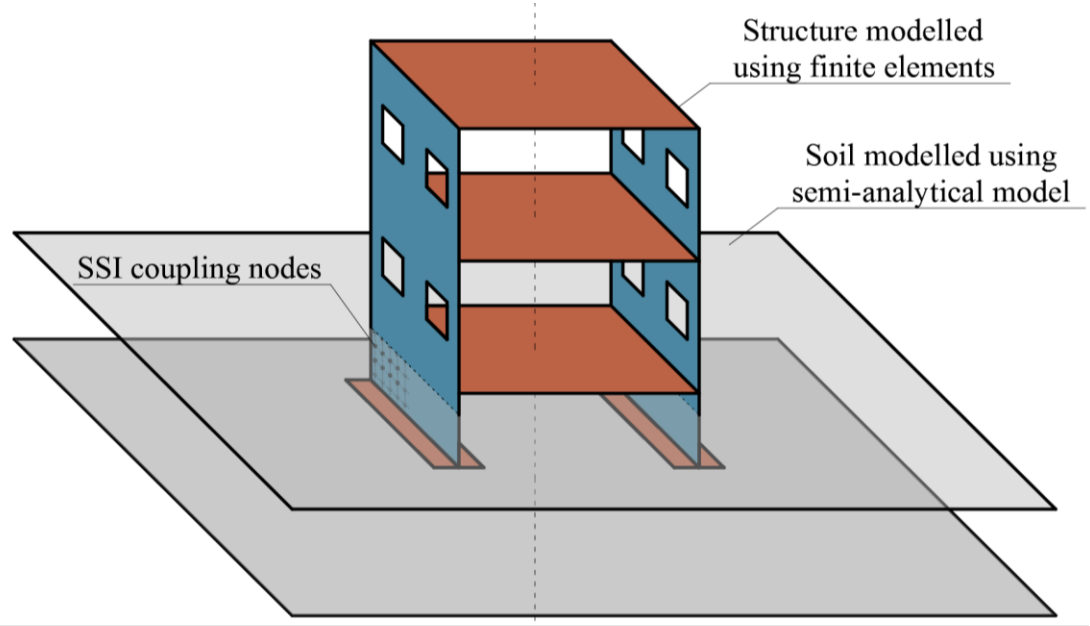
- Thomson and Haskell Transfer matrix method is used
- Flexibility approach is used to assemble the system
- Additional numerical stabilization technique by Wang



Semi-analytical soil: coupling to FE

Coupling structures to the soil

- The structures are modelled using the finite element method (FEM)
- Beam, shell, solid elements can be used
- Finite element structures are connected to the soil through structure-soil interaction (SSI) nodes



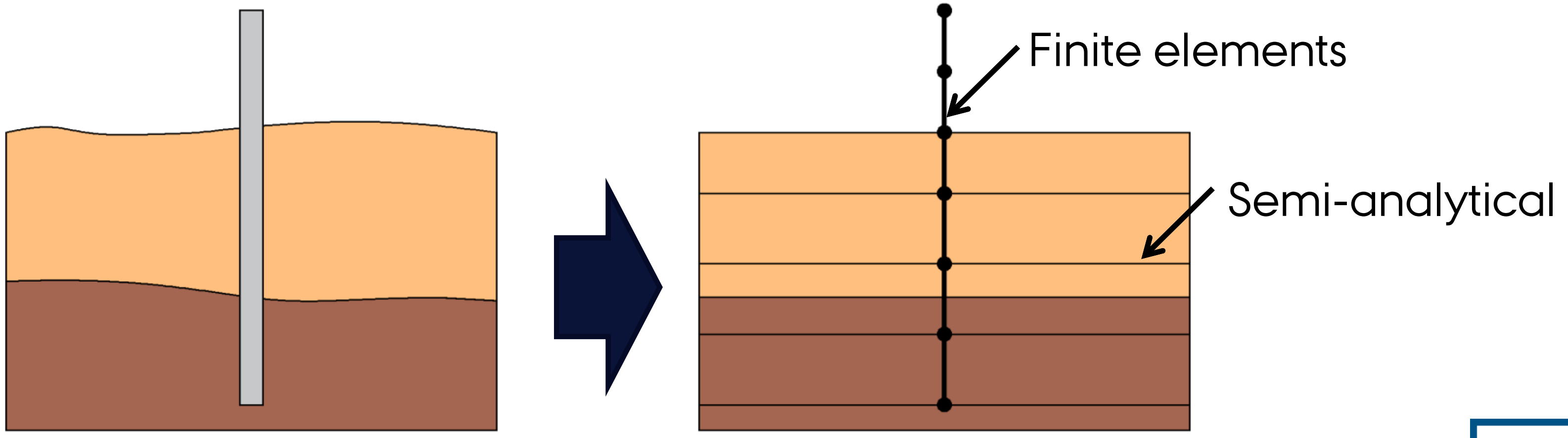
Semi-analytical soil: coupling to FE

Solution is modified to include embedded structures:

- New layer interfaces are created for depths where SSI nodes are present
- A global flexibility matrix is established including all SSI nodes
- Inversion of flexibility matrix provides soil dynamic stiffness matrix

To couple the solution to finite elements:

- FE matrices are transformed to dynamic stiffness matrix
- Dynamic stiffness matrices for the structure and soil are added together



Real system Simplified system

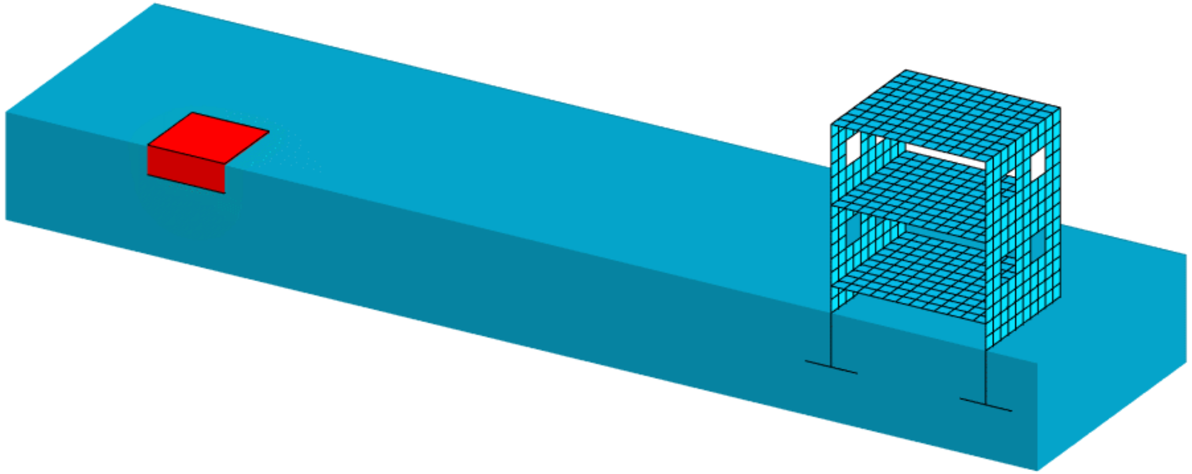
Semi-analytical soil: coupling to FE

Advantages:

- Computationally efficient
- Structures with complex geometry can be modelled
- Fully coupled structure-soil system in 3 dimensions

Disadvantages:

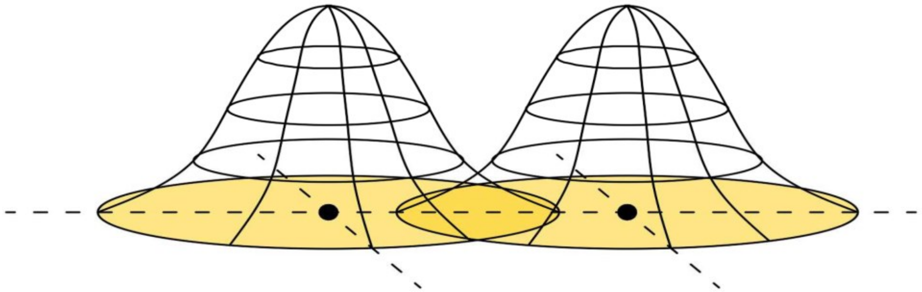
- Only frequency domain solution available
- Complex convergence procedure

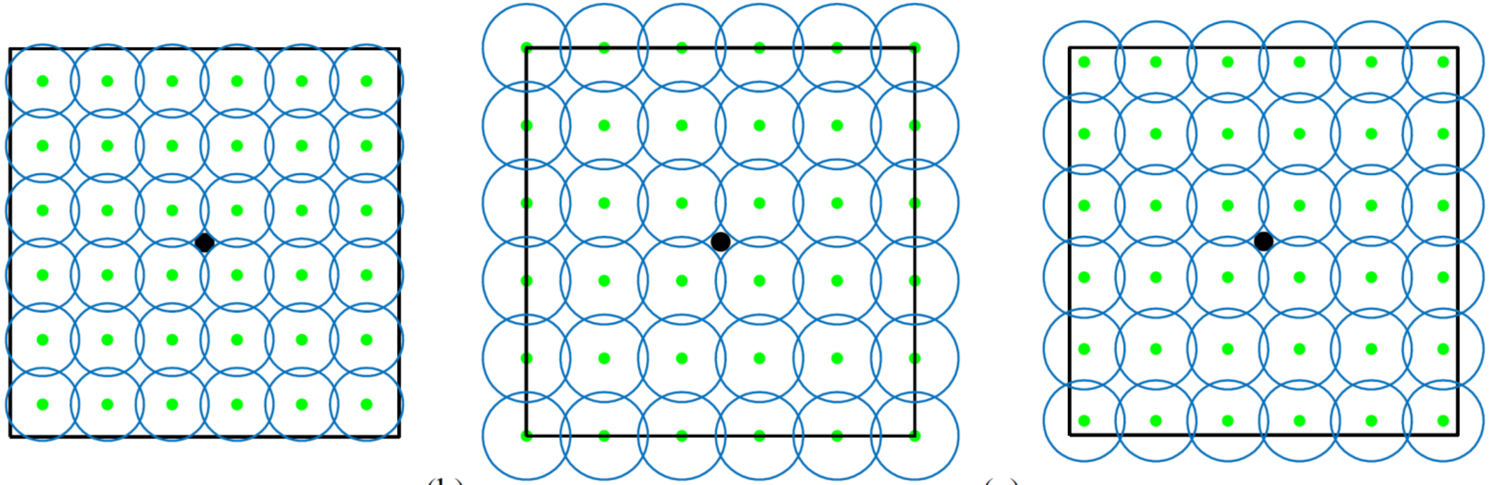


Dynamic soil–structure interaction

Modelling of rigid structures

- Applicable to a wide range of cases
- Easy implementation
- Computationally efficient and accurate solution
- Difficult discretization

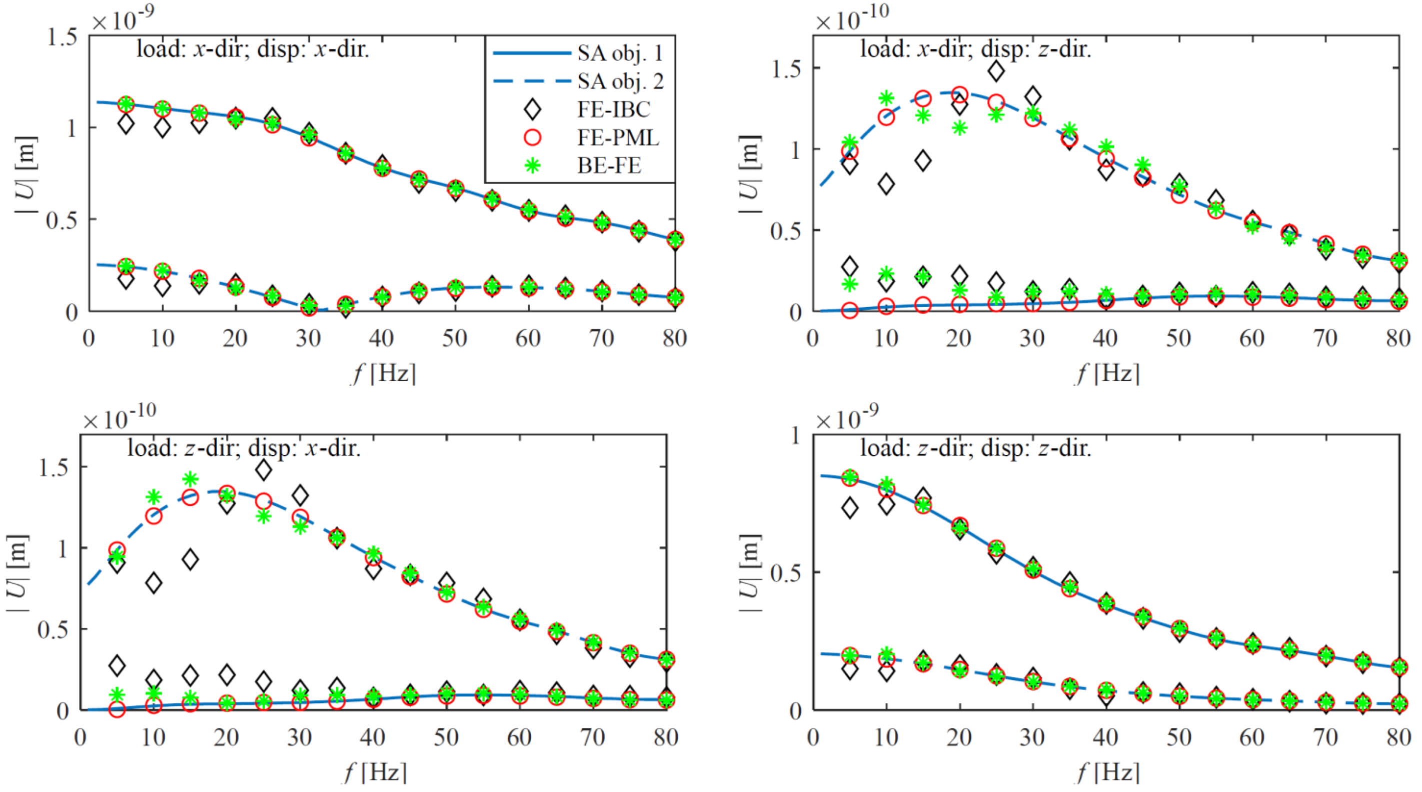




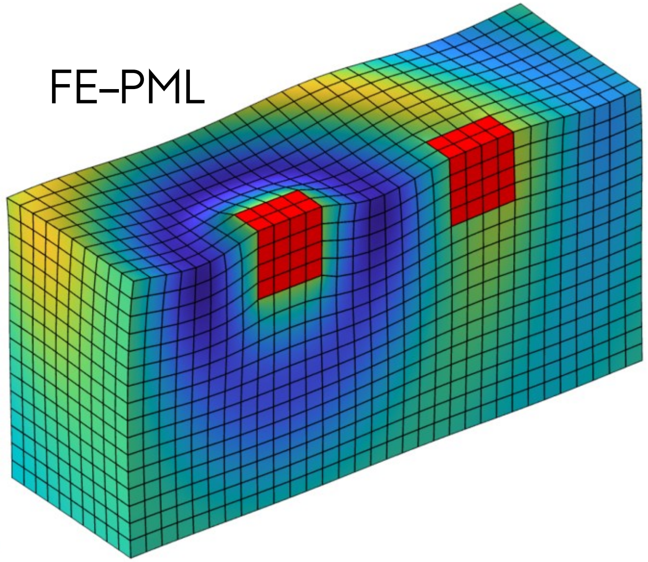
(a) (b) (c)

Dynamic soil–structure interaction

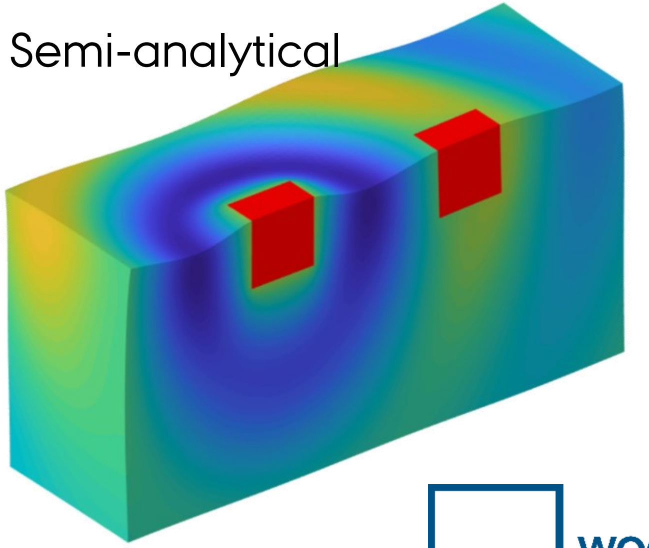
Three-dimensional rigid blocks



FE–PML

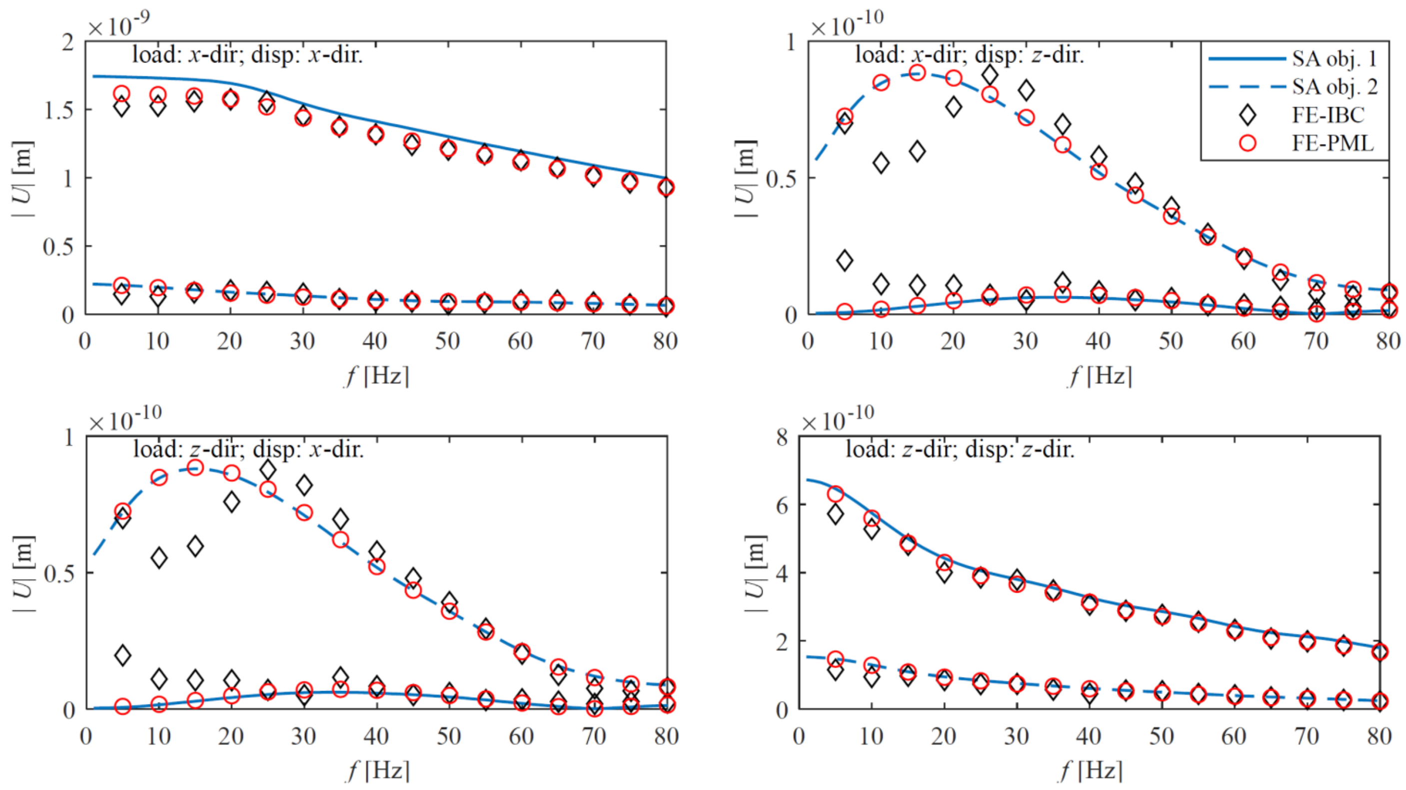


Semi-analytical

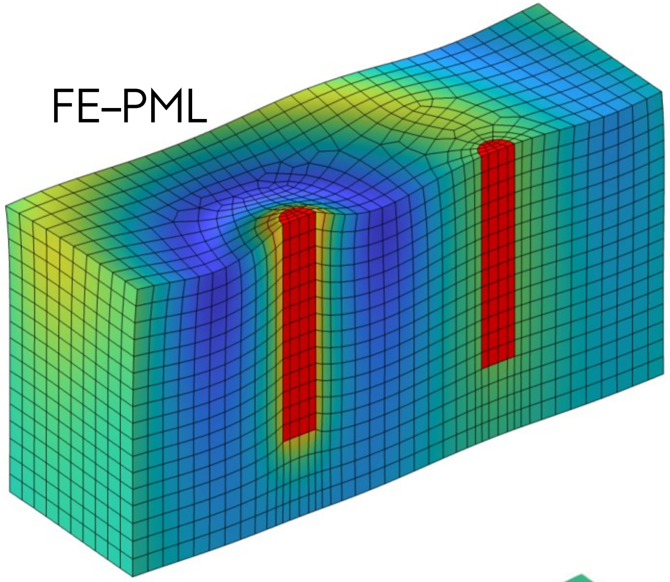


Dynamic soil–structure interaction

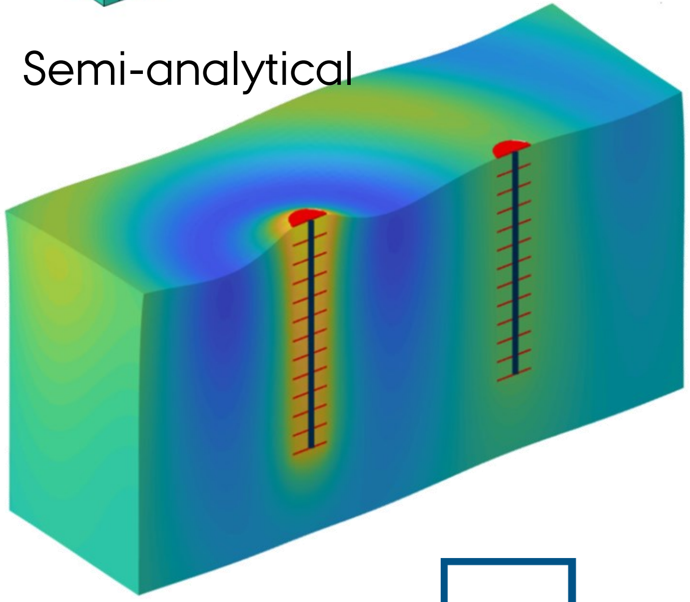
Validation case: pile modelling



FE–PML



Semi-analytical

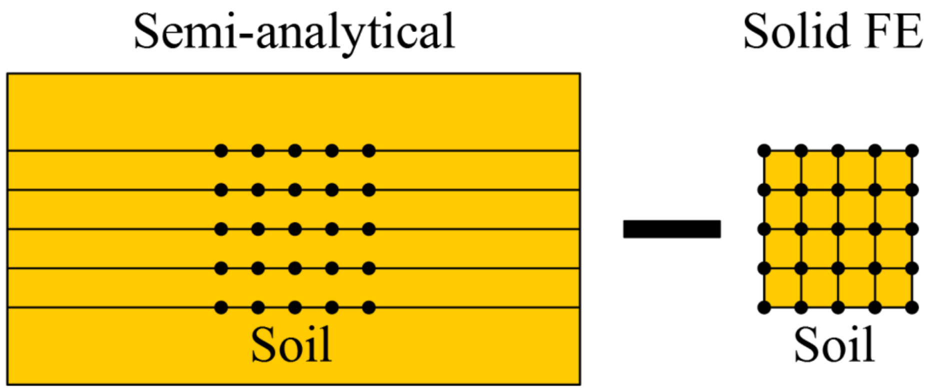


Dynamic soil–structure interaction

Modelling cavities inside the soil.

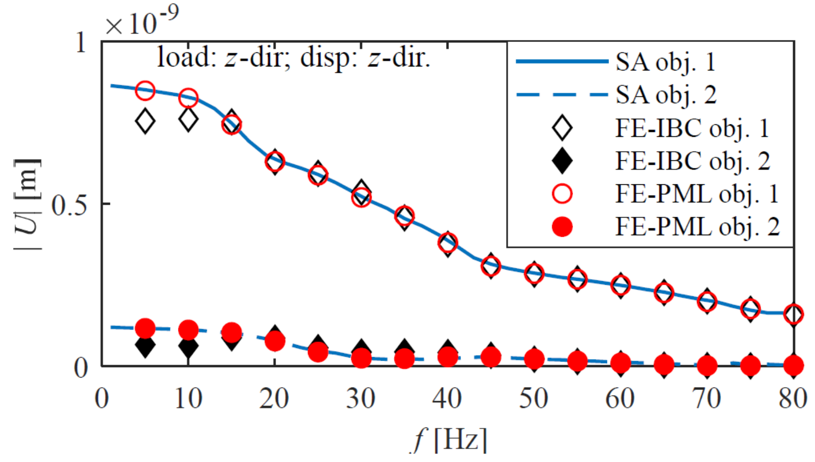
- Direct inclusion into the semi-analytical approach is not possible
- Cavity is discretized using solid FE elements, with properties of the soil
- The FE dynamic stiffness matrix is subtracted from the semi-analytical solution

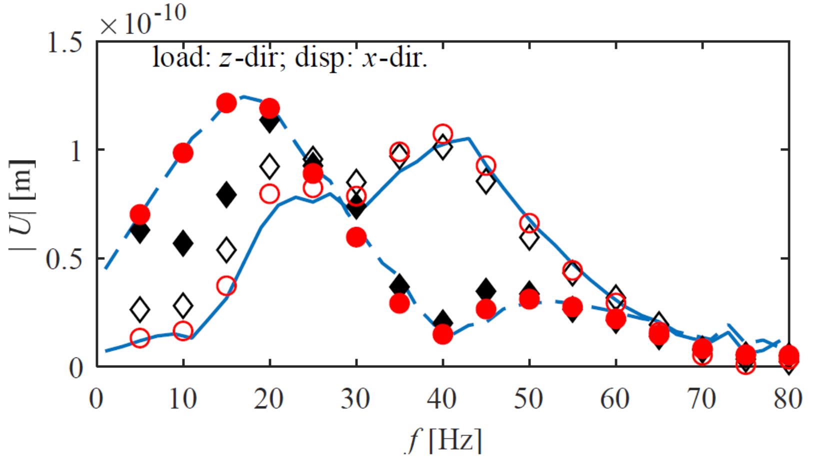
$$\mathbf{D}_{\mathrm{RC}}(\omega) = \begin{bmatrix} \mathbf{D}_{\mathrm{R}}^{\mathrm{rr}}(\omega) & \mathbf{D}_{\mathrm{R}}^{\mathrm{rc}}(\omega) \\ \mathbf{D}_{\mathrm{R}}^{\mathrm{cr}}(\omega) & \mathbf{D}_{\mathrm{R}}^{\mathrm{cc}}(\omega) - \mathbf{D}_{\mathrm{C}}(\omega) \end{bmatrix}$$



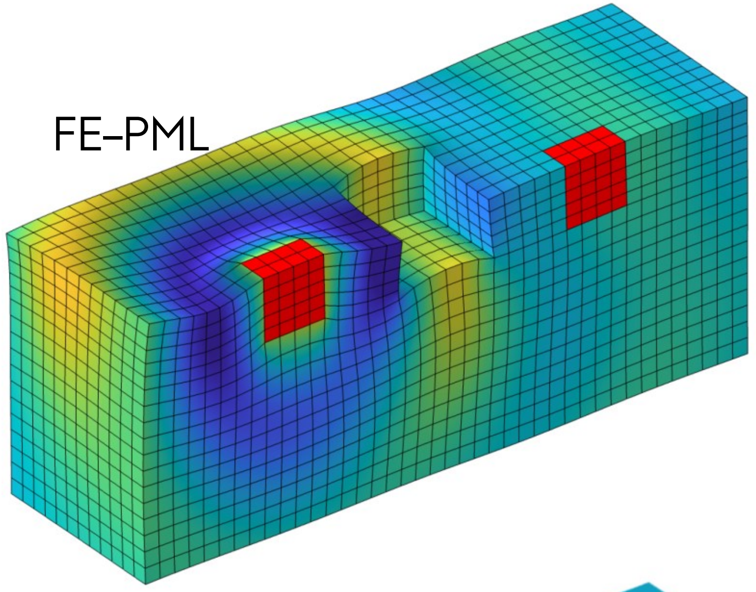
Dynamic soil–structure interaction

Validation case: cavity modelling

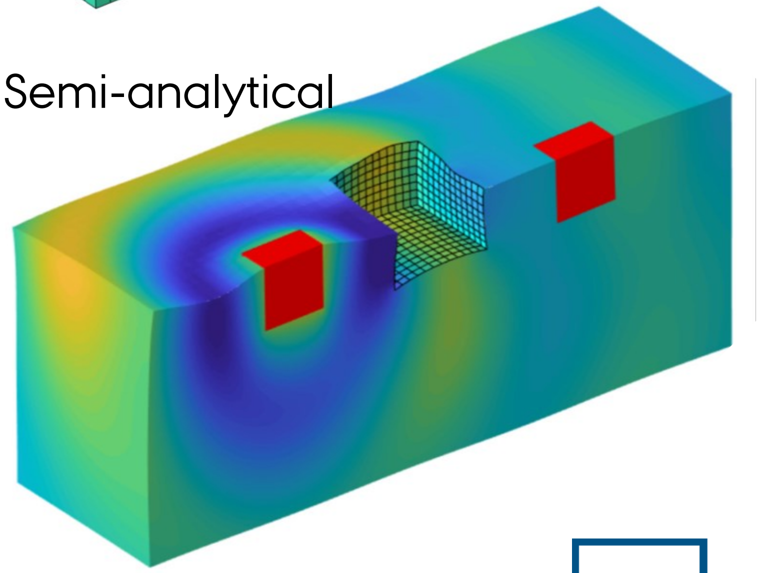




FE–PML



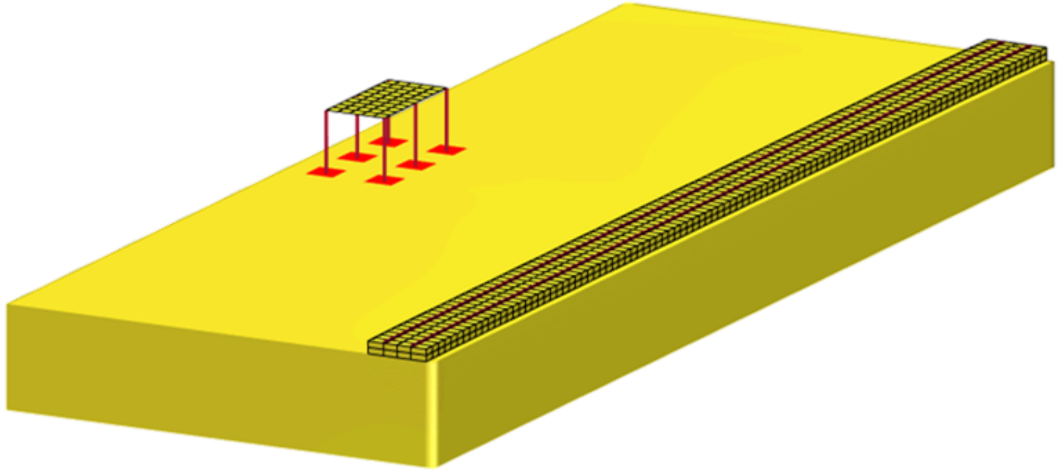
Semi-analytical

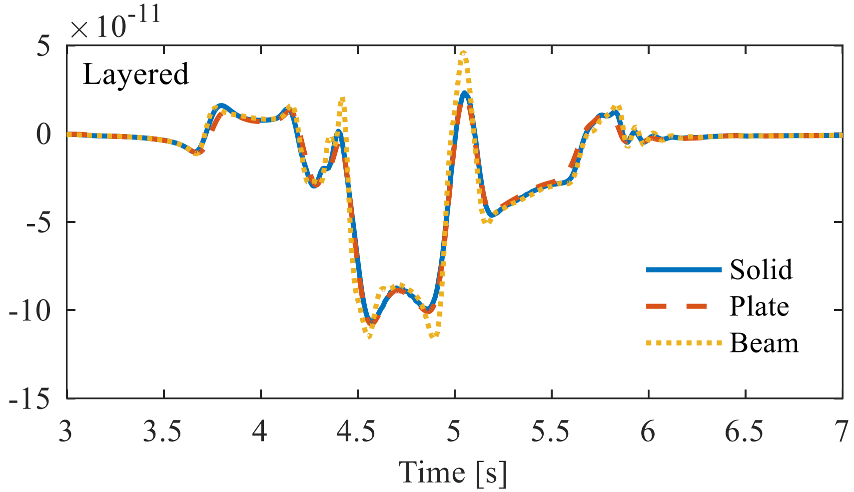


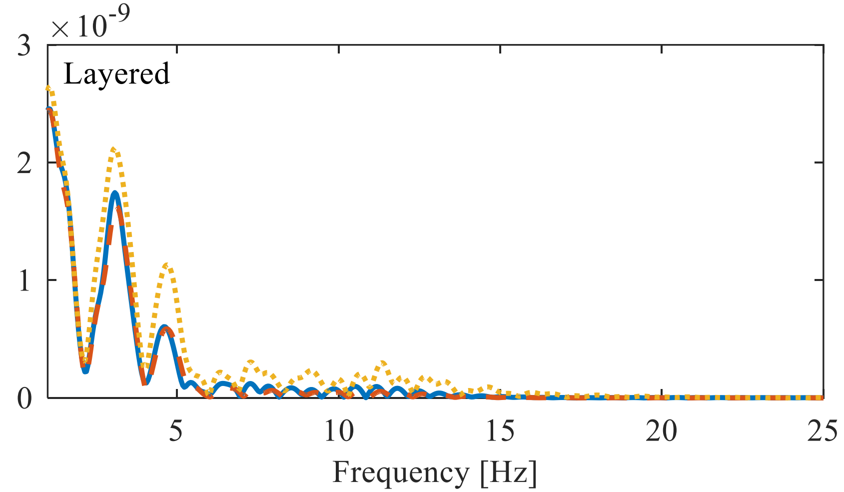
Railway vibration modelling

Fixed frame of reference model

- Moving load
- Load moving with constant 40 m/s speed
- IFT used to obtain time domain response
- 1024 discrete frequencies in 0-100 Hz range







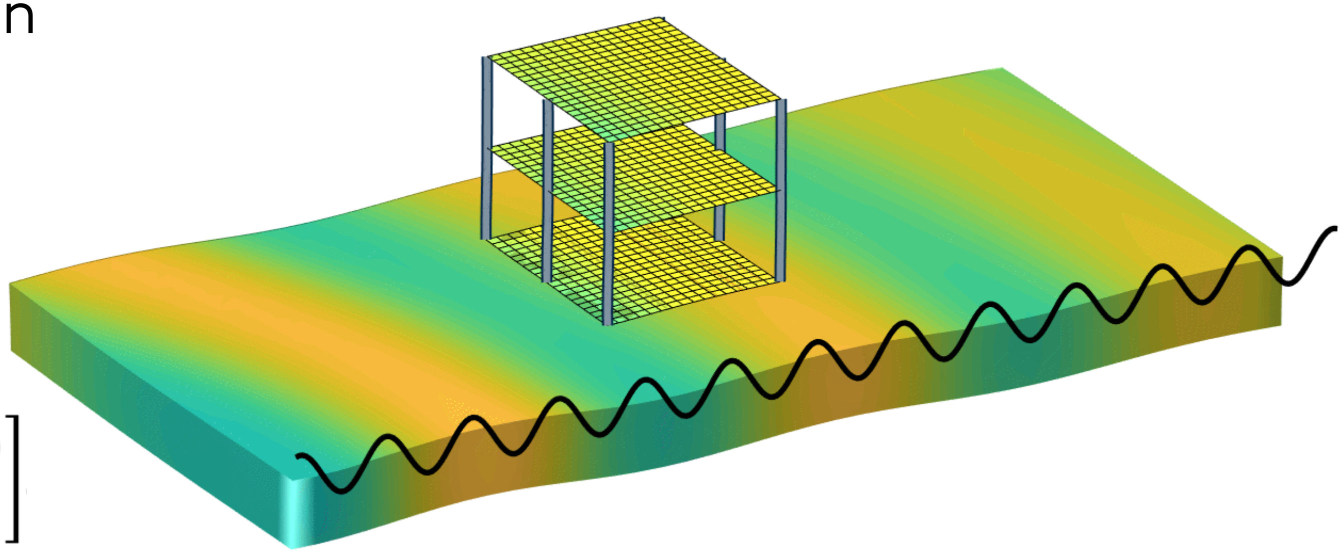
Railway vibration modelling

Mixed frame of reference approach

- Semi-analytical approach is used in both moving and fixed frame of reference (FOR)
- FORs are coupled via analytically derived coupling terms

$$\tilde{\mathbf{R}}_{\mathrm{g}}(\omega_{\mathrm{m}},\omega_{\mathrm{f}}) = \begin{bmatrix} \mathbf{R}_{\mathrm{rr}}(\omega_{\mathrm{m}}) & \tilde{\mathbf{R}}_{\mathrm{rs}}(\omega_{\mathrm{f}},\omega_{\mathrm{m}}) \\ \tilde{\mathbf{R}}_{\mathrm{sr}}(\omega_{\mathrm{m}},\omega_{\mathrm{f}}) & \mathbf{R}_{\mathrm{ss}}(\omega_{\mathrm{f}}) \end{bmatrix}$$

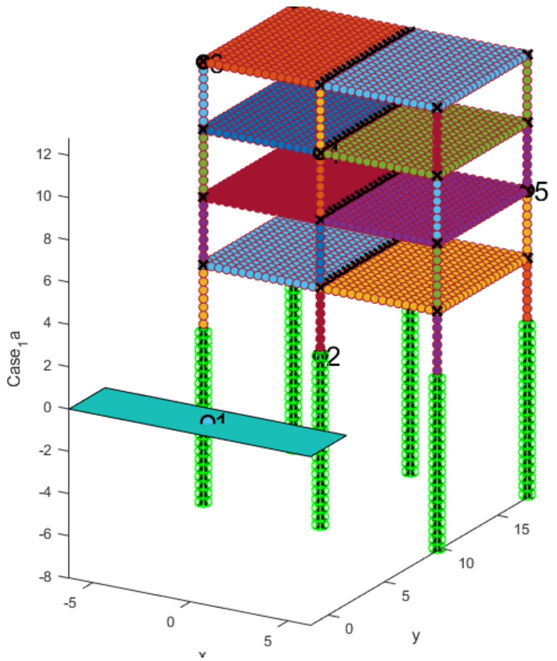
- Partially coupled and fully coupled solution procedures are proposed

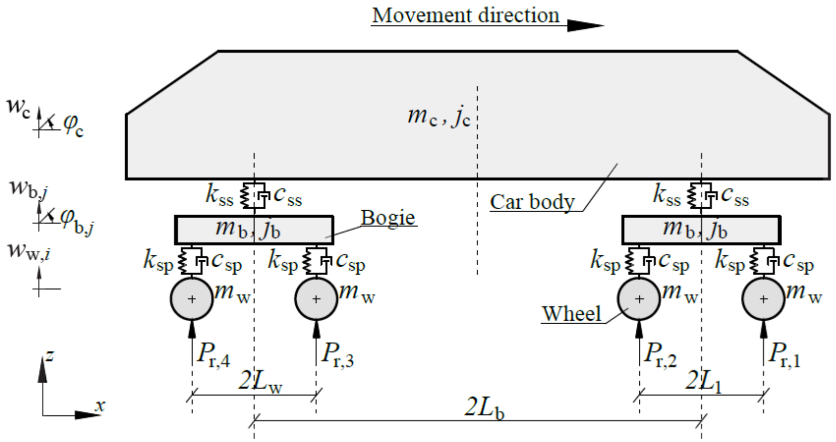


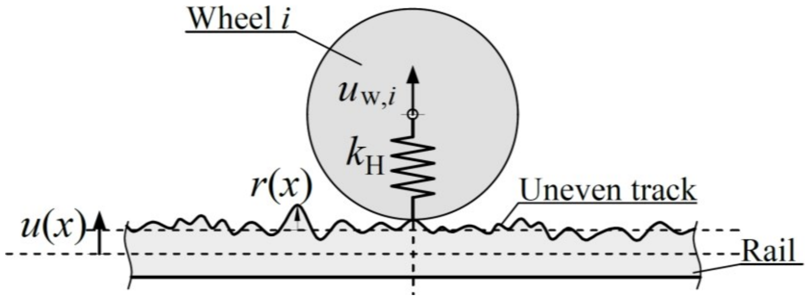
Railway vibration modelling

Mixed frame of reference approach

- Analytical railway track model in a moving frame of reference
- Vehicle modelled as a multi-body system
- System excited by track irregularities
- Single-step solution for the full system
- FE structures can be easily added to the system in both FORs
- Possible to directly model rigid objects interacting with soil







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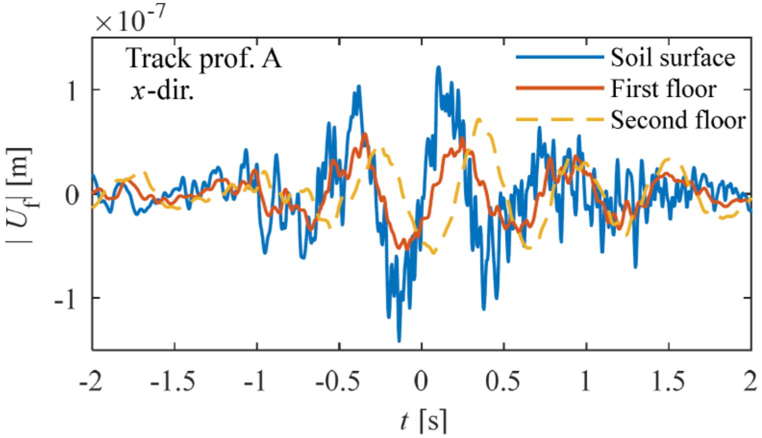
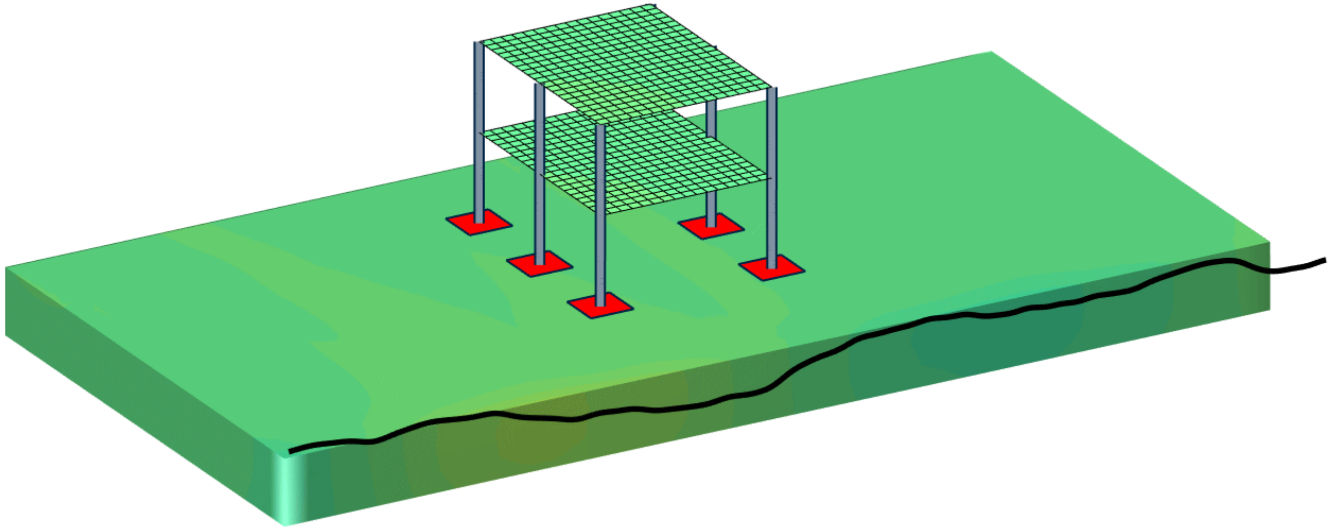
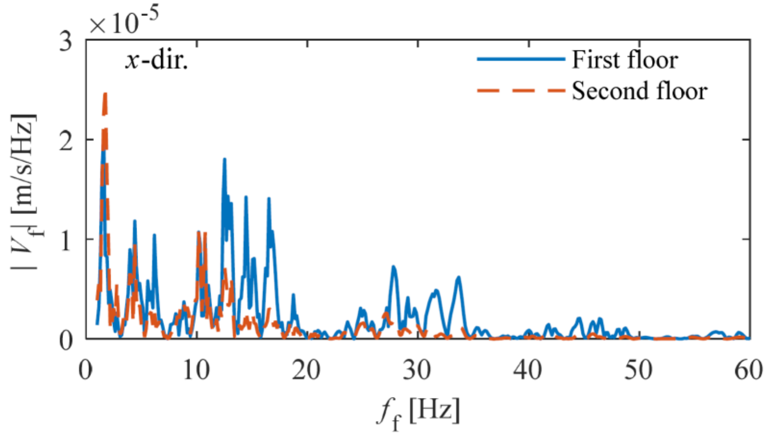
Railway vibration modelling

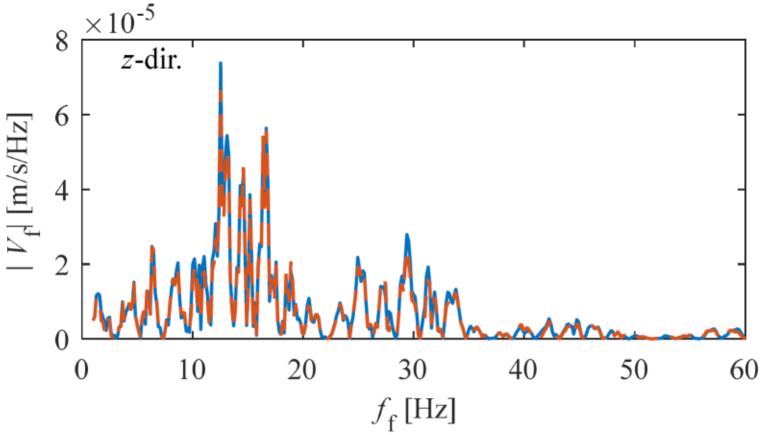
Mixed frame of reference approach

- Partially coupled system





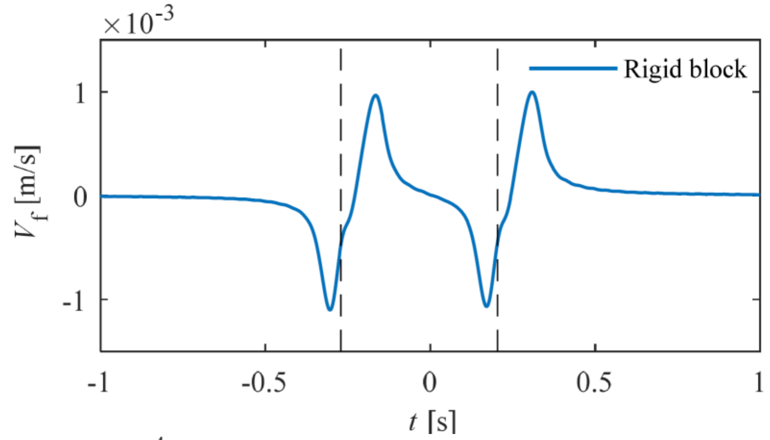


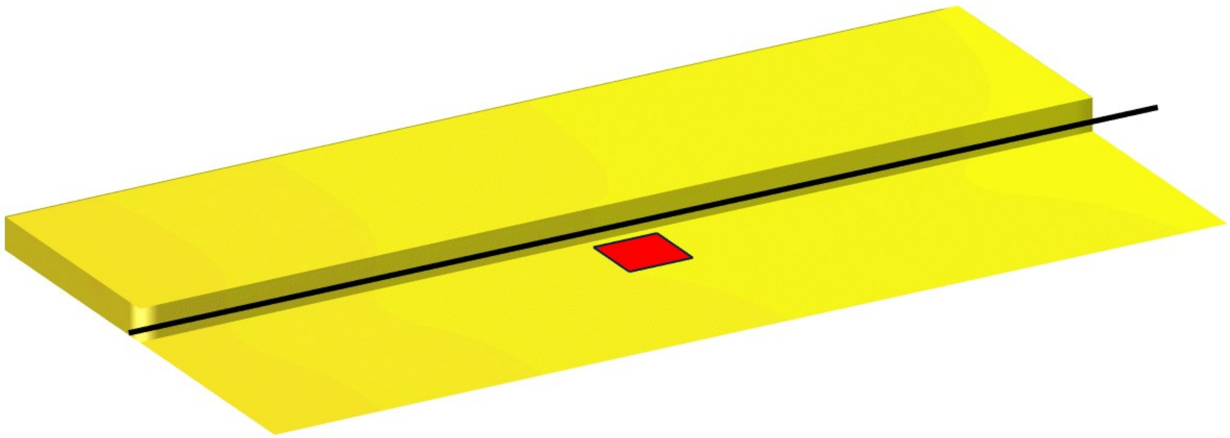


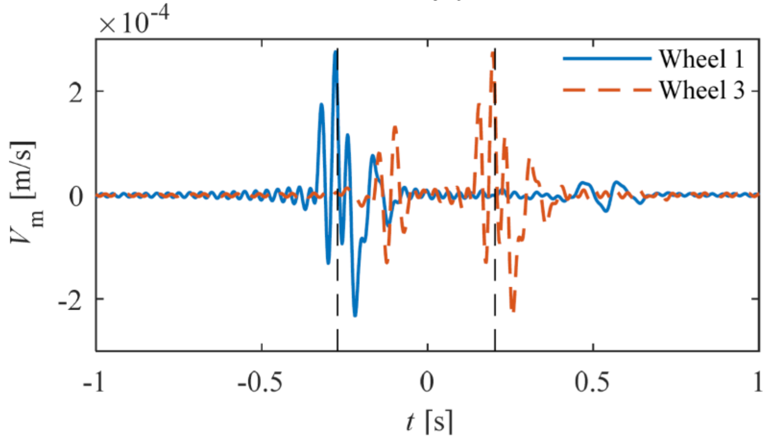
Railway vibration modelling

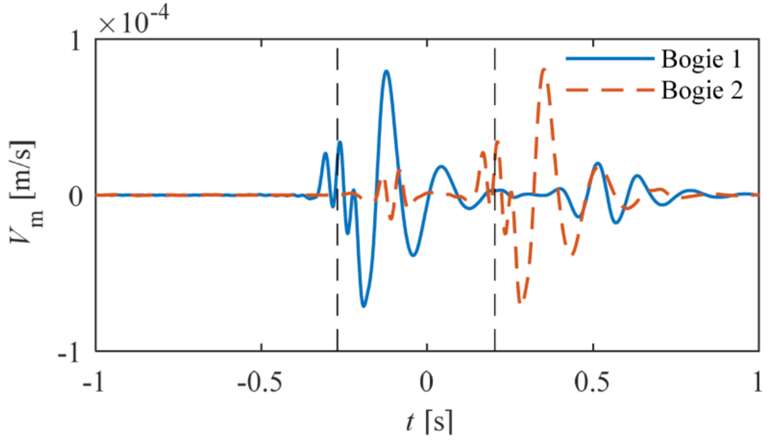
Mixed frame of reference approach

- Fully coupled system









Conclusions

- The semi-analytical method proved to be a versatile approach
- Numerically efficient and precise solution is possible with combination with FE
- Mixed frame of reference approach expands the range of problems that can be modelled using the semi-analytical soil model

References and links

References:

- P. Bucinskas, Propagation and Effects of Vibrations in Densely Populated Urban Environments, PhD dissertation, 2020, ISBN 978-87-7507-489-1, https://doi.org/10.7146/aul.389
- P. Bucinskas, E. Ntotsios, D.J. Thompson and L.V. Andersen, Modelling train-induced vibration of structures using a mixed-frame-of-reference approach. Journal of Sound and Vibration, Volume 491, 2021, 115575, ISSN 0022-460X, https://doi.org/10.1016/j.jsv.2020.115575

Computational models:

- Semi-analytical model with rigid objects
  https://drive.google.com/open?id=1a_akCbuZU1H1hX_KghWxy1drwMIb9Yuk
- Semi-analytical model with FE structures
  https://drive.google.com/open?id=1UndTmhjeAuRBgKCKqMQkuiKx-MRawB04

Thank you for your attention!

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