Complex Geometry at Aarhus University
Abstracts

Aarhus

February 19-21, 2025

Daniele Angella

The Chern-Ricci flow on Inoue-Bombieri surfaces

In the attempt to move from the Kähler to the non-Kähler setting, one can formulate several problems concerning Hermitian metrics on complex manifolds with special curvature properties. Among these problems, we highlight the existence of Hermitian metrics with constant scalar curvature with respect to the Chern connection and the generalizations of the Kähler-Einstein condition to the non-Kähler setting. These problems are typically translated into analytic PDEs. In this context, the Chern-Ricci flow plays a significant role. The Chern-Ricci flow is a parabolic evolution equation for Hermitian metrics that generalizes the Kähler-Ricci flow to Hermitian manifolds. The behavior of solutions to the Chern-Ricci flow is expected to reflect the underlying complex structure. In particular, understanding the behaviour of the Chern-Ricci flow on non-Kähler compact complex surfaces is of special interest, as minimal class VII surfaces have not yet been fully classified. In this talk, we address the problem of convergence of the normalized Chern-Ricci flow on Inoue-Bombieri surfaces starting at Gauduchon metrics. This talk is based on joint works with Valentino Tosatti and Mauricio Corrêa, and inspired by collaborations and discussions with Simone Calamai, Francesco Pediconi, Cristiano Spotti, Oluwagbenga Joshua Windare, and many others.

Beatrice Brienza

Bismut Hermitian Einstein and strong HKT manifolds

A manifold (M, J_1, J_2, J_3) is called hypercomplex if each J_i is a complex structure and $\{J_1, J_2, J_3\}$ satisfy the quaternionic relations. A quaternionic Hermitian metric g is called HKT (hyper-Kähler with torsion) if $\nabla_1^B = \nabla_2^B = \nabla_3^B =: \nabla^B$. If, in addition, the Bismut torsion of ∇^B is closed, the metric is called strong HKT. Whenever we have a strong HKT metric, the three Hermitian structures (J_i, g) are Bismut Hermitian Einstein, namely, they are pluriclosed and their Bismut Ricci curvature vanishes. In this talk, we will discuss some properties and examples of compact strong HKT and, more generally, BHE manifolds. This is part of a joint work with A. Fino and G. Grantcharov and a work in progress with A. Fino, G. Grantcharov, and M. Verbitsky.

Ionut Chiose

Special metrics on compact complex manifolds

On a compact Kähler manifold, many wonderful things happen. However, there are many classes of compact complex manifolds that do not support Kähler metrics, but instead they support some *special* metrics - balanced, SKT, astheno-Kähler etc.. In this talk, we describe some properties of these manifolds, and discuss some conjectures.

Joana Cirici

Batalin-Vilkovisky and hypercommutative algebras in complex geometry

I will review some constructions of BV and hypercommutative algebras for manifolds with additional geometric structures, ranging from Poisson to Hermitian manifolds. Such algebra structures are related to the extended deformation theory introduced by Barannikov and Kontsevich for Calabi-Yau manifolds. I will explain how, using mixed Hodge theory at the homotopical level, one can prove hypercommutative formality of compact Kähler manifolds. This talk includes joint results with Geoffroy Horel and with Scott Wilson.

Raul Gonzales-Molina

Pluriclosed flow and the Hull-Strominger system

We define an extension of pluriclosed flow aiming at constructing solutions of the Hull-Strominger system. Interestingly, this flow admits various natural geometric formulations: the evolution equations stem from the theory of string algebroids, a class of Courant algebroids which occur naturally in higher gauge theory and heterotic string theory. Using these, we interpret the flow as generalized Ricci flow and also as a higher/coupled version of Hermitian-Yang-Mills flow. Moreover, under natural symmetry reduction ansatze we are able to carry some of the techniques of generalized Ricci flow to our context. Regarding our main analytical results, we prove smooth regularity for the Hull-Strominger system as a consequence of a priori estimates for uniformly parabolic solutions, and show existence and convergence results for the flow on special backgrounds. Time permitting, we explore the application of the flow to discuss a conjectural relationship to the geometrization of Reid's fantasy. This talk is based on joint work with Mario Garcia-Fernandez and Jeffrey Streets (arXiv:2408.11674).

Adela Latorre

Nilmanifolds with non-nilpotent complex structures and their pseudo-Kähler geometry

A nilmanifold $N = \Gamma/G$ is a compact quotient of a connected, simply connected, nilpotent Lie group G by a discrete subgroup Γ. Although finding a complex structure J on a real nilmanifold N is in general not easy, the task can be slightly simplified when one focuses on complex structures of invariant type. An invariant complex structure on N is a complex structure coming from an endomorphism $J : \mathfrak{g} \to \mathfrak{g}$ of the Lie algebra $\mathfrak{g}$ of G satisfying $J^2 = -Id$ and $N_J = 0$, where N_J denotes the Nijenhuis tensor of J. These types of complex structures can be classified into nilpotent and non-nilpotent, depending on the behaviour of the so-called ascending J-compatible series $\{\mathfrak{a}_k(J)\}_k$ of (g, J). In this talk, we will focus on the non-nilpotent case. We will present the classification of non-nilpotent complex structures on 8-dimensional nilpotent Lie algebras and then apply it to study pseudo-Kähler geometry.

Hoang Chinh Lu

Complex Monge-Ampère mean field equations

We establish the uniqueness of solutions to complex Monge-Ampère mean field equations when the temperature parameter is small. In the local setting of bounded hyperconvex domains, our result partially confirms a conjecture by Berman and Berndtsson. Our approach also extends to the global context of compact complex manifolds. This is a joint work with T.T. Phung arxiv:2501.18281.

Asia Mainenti

Hodge-Riemann balanced structures on solvmanifolds

A Hodge-Riemann balanced structure on a complex manifold is the datum of a balanced metric whose $(n-1)$-th power can be decomposed into the wedge product of two differential forms, satisfying the classical Hodge-Riemann bilinear relations. Such structures were introduced by X. Chen and R. Wentworth, to generalize the nonabelian Hodge correspondence to the non-Kähler setting. However, there are no known examples of Hodge-Riemann balanced structures on non-Kähler manifolds. The aim of this talk is to address this lack of examples, highlighting the relation with p-Kähler structures and discuss some obstruction results in the class of solvmanifolds. Lastly, we will present the first example of such a structure on a non-Kähler, non-compact complex manifold obtained as the product of the Iwasawa manifold by C. This is joint work with A. Fino.

Andrei Moroianu

Quaternion-Kähler manifolds with non-negative quaternionic curvature

Compact Hermitian symmetric spaces are Kähler manifolds with constant scalar curvature and non-negative sectional curvature. A famous result by A. Gray states that, conversely, a compact simply connected Kähler manifold with constant scalar curvature and non-negative sectional curvature is a Hermitian symmetric space. The aim of the present talk is to transpose Gray's result to the quaternion-Kähler setting. In order to achieve this, we introduce the quaternionic sectional curvature of quaternion-Kähler manifolds, we show that every Wolf space has non-negative quaternionic sectional curvature, and we prove that, conversely, every quaternion-Kähler manifold with non-negative quaternionic sectional curvature is a Wolf space. The proof makes crucial use of the nearly Kähler twistor spaces of positive quaternion-Kähler manifolds. The talk is based on joint work with Uwe Semmelmann and Gregor Weingart.

David Pectu

A generalization of Inoue surfaces S^+

We begin the talk with the construction of a class of compact complex manifolds that are higher dimensional analogues of Inoue surfaces of type S^+. Then, we discuss some of the topological and analytical properties of these objects, showing that, for a generic choice of the data used in the construction, they do not contain hypersurfaces. We establish that in dimensions higher than two, these manifolds are not locally conformally Kähler. Additionally, we describe a left-invariant metric that is locally conformally balanced. In fact, this metric is the natural generalization of the Tricerri metric on surfaces of type S^+.

Jonas Stelzig

Higher operations in complex and Kähler geometry

In recent years, higher cohomological operations have emerged in complex geometry, akin to classical Massey products from topology, but sensitive to the complex structure, eg in works of Deninger, Angella and Tomassini, and Tardini. We discuss some context from rational homotopy theory and why they are interesting. We then focus on the Kähler case: Unlike classical Massey products, these holomorphic operations can be nontrivial on compact Kähler manifolds. On the other hand, they necessarily vanish on toric manifolds and on compact homogeneous Kähler manifolds. Joint work with Aleks Milivojevic, Giovanni Placini and Leo Zoller.

Jeffrey Streets

Non-Kähler Calabi-Yau geometries

I will discuss classification and rigidity results for non-Kähler Calabi-Yau geometries in low dimensions. The main result is that in complex dimension 3 all such are Bismut-flat. The proof involves finding an a priori symmetry, and then classifying the reduced equations, a special case of the Hull-Strominger system. Joint work with V. Apostolov, G. Barbaro, K.H. Lee.

Alejandro Tolcachier

Six-dimensional complex solvmanifolds with non-invariant trivializing sections of their canonical bundle

It is well known, thanks to independent results of Cavalcanti-Gualtieri and Barberis-Dotti-Verbitsky, that every nilmanifold equipped with an invariant complex structure has holomorphically trivial canonical bundle, as there exists an invariant holomorphic trivializing section. In contrast, for complex solvmanifolds such a section may not exist. Moreover, in a recent work with A. Andrada, we provided examples of complex solvmanifolds that admit a trivializing holomorphic section of its canonical bundle which is not invariant under the Lie group action. In this talk, I will present the classification of six-dimensional solvable strongly unimodular Lie algebras corresponding to such complex solvmanifolds, thus extending the earlier results of Fino, Otal and Ugarte for the invariant case. In particular, I will show a new example of a solvable Lie algebra admitting a complex structure with a holomorphic $(3,0)$-form, and an example of a complex solvmanifold with holomorphically trivial canonical bundle that is not biholomorphic to any complex solvmanifold with an invariant trivializing section of its canonical bundle. This talk is based on the preprint https://arxiv.org/abs/2412.02325.