

The Langlands philosophy postulates a correspondence between certain automorphic representations of $GL(n)$, and (compatible systems of) certain n -dimensional Galois representations. Congruences of automorphic forms can then be modeled by p -adic families of Galois representations, and one can ask how Galois theoretic constraints describe spaces of automorphic representations that reduce to the same mod p Galois representation. This elementary minicourse gives an introduction to the deformation theory of Galois representations in the sense of Mazur and its relevance to modular Galois representations. Topics are: (1) Galois representations and examples from geometry. (2) Deformation functors and universal rings. (3) Local and global deformation problems. And if time permits: (4) An introduction to Ramakrishna's lifting method.

References:

[Böckle, Gebhard](#) Deformations of Galois representations. *Elliptic curves, Hilbert modular forms and Galois deformations*, 21–115, [Adv. Courses Math. CRM Barcelona](#), Birkhäuser/Springer, Basel, 2013.

[Gouvêa, Fernando Q.](#) Deformations of Galois representations. Appendix 1 by Mark Dickinson, Appendix 2 by Tom Weston and Appendix 3 by Matthew Emerton. [IAS/Park City Math. Ser., 9](#), *Arithmetic algebraic geometry (Park City, UT, 1999)*, 233–406, *Amer. Math. Soc.*, Providence, RI, 2001.

[Mazur, B.](#) Deforming Galois representations. *Galois groups over $\mathbb{Q}$* (Berkeley, CA, 1987), 385–437, [Math. Sci. Res. Inst. Publ., 16](#), Springer, New York, 1989.

[Mazur, Barry](#) An introduction to the deformation theory of Galois representations. *Modular forms and Fermat's last theorem (Boston, MA, 1995)*, 243–311, Springer, New York, 1997.

[Neukirch, Jürgen](#); [Schmidt, Alexander](#); [Wingberg, Kay](#) Cohomology of number fields. Second edition. [Grundlehren der mathematischen Wissenschaften \[Fundamental Principles of Mathematical Sciences\], 323](#). Springer-Verlag, Berlin, 2008.

[Shotton, Jack](#) The Breuil-Mézard conjecture when $l \neq p$. [Duke Math. J.](#) **167** (2018), no. 4, 603–678.

[Taylor, Richard](#) On icosahedral Artin representations. II. [Amer. J. Math.](#) **125** (2003), no. 3, 549–566.