Abstracts for 3 hour workshops

Monday July 14

13:30-16:30: Workshops 3 hours: **Theme 3**

Janet Heine Barnett (USA) **Room A303**: *Abstract Awakenings in Algebra: A Guided Reading Approach to Teaching Modern Algebra Via Original Sources*.

This workshop will explore a particular approach to the use of original historical sources in the undergraduate university mathematics classroom. The cornerstone of this approach is the extensive use of excerpts from original source material close to or representing the discovery of key mathematical concepts and theory as a means to develop the material in question. Through guided reading of the excerpts and completion of exercises based upon them, students are prompted to explore these ideas and develop their own understanding of them. Each excerpt is introduced by brief historical comments and biographical information about its author to set it in context and offer students a view of the humanistic aspect of mathematics. By placing a problem in its historical context, the student is also able to follow the thought process that led to the discovery of complicated and subtle mathematical concepts while simultaneously being guided to construct their own understanding of the present day ideas evolving via the project exercises.

Examples of this approach are found in a compendium of projects developed and tested since 2008 by an interdisciplinary team of mathematicians and computer scientists for the teaching of topics in discrete mathematics with support from the US National Science Foundation. All our projects are available via our two web resources (http://www.cs.nmsu.edu/historical-projects/ , http://www.math.nmsu.edu/hist_projects/).

More specifically, the workshop will focus on student projects designed for use in a first course in Abstract Algebra in the standard US undergraduate curriculum, including portions of the project *Abstract awakenings in algebra: Early group theory in the works of Lagrange, Cauchy, and Cayley* (project # 11 at http://www.cs.nmsu.edu/historical-projects/). Successfully tested as a textbook replacement for a significant portion of an abstract algebra course at three US institutions to date, this project exemplifies the benefits of using select original source excerpts to draw attention to mathematical subtleties which modern texts may take for granted. In keeping with the historical record, this often leads to a more concrete approach than is typical of current texts. For example, to provide context for the abstract group concept first defined in an 1854 paper by Arthur Cayley, this project begins by studying specific mathematical systems (e.g., roots of unity in Lagrange, permutations groups in Cauchy) which were well-developed prior to Cayley's explicit recognition of their common structure. Cayley's own references to these and other specific nineteenth century appearances of the group concept render his paper a powerful lens on the process mathematical abstraction that more standard textbooks do not provide.

Additionally, workshop participants will explore portions of a student project in ring theory, based on excerpts from works by Dedekind, Fraenkel, E. Noether and Krull, which is currently under development for use in a first course on abstract algebra. The workshop agenda will also include a discussion of implementation and evaluation issues, an overview of the general pedagogical goals guiding our work, and analysis of specific features of these particular abstract algebra projects relative to those goals and other theoretical frameworks.

Anne Boyé, Annie Michel-Pajus and Martine Bühler (France) **Room A210**: *Algorithmes: l'apport des textes historiques en classe. Algorithms: an approach based on historical texts in the classroom.*

Les nouveaux programmes de mathématiques pour les lycées en France, mettant en lumière l'importance des algorithmes en mathématiques, insistent sur la formation à une sorte de « pensée algorithmique ». Il est demandé explicitement d'étudier et de construire, en classe, des algorithmes plus ou moins élémentaires, dans de multiples situations.
Notre atelier se propose d'étudier comment ces travaux, en classe de mathématiques, pourraient être éclairés par l'utilisation de sources historiques, et pas seulement par l'utilisation des ordinateurs.
Nous commencerons par un panorama historique, illustré d'extraits de textes, portant aussi bien sur l'origine et le sens du mot « algorithme », que sur les algorithmes effectivement rencontrés dans l'histoire.
Nous montrerons ensuite comment l'étude en classe de textes historiques, mettant l'accent sur le côté algorithmique des travaux de certains mathématiciens, permet aux élèves de mieux saisir certains concepts mathématiques, par exemple dans le cas des suites, en particulier celles définies par récurrence.
Nous présenterons des séances expérimentées en classe de première et terminale scientifiques (élèves de 16 à 18 ans), construites autour de textes extraits des *Métriques* de Héron d'Alexandrie, de l'*Algebra* de Bombelli et de Fermat.
Les textes seront présentés à la fois en anglais et en français.
The new curriculum in French High schools, which is currently being implemented, highlights the importance of algorithms in mathematics, promoting a kind of « algorithmic way of thinking ». It explicitly requires that elementary but varied works on algorithms be carried out in the classroom.
In the workshop, we'll attempt to show that this work could be carried out on the basis of historical sources, and not only with a computer.
We'll begin with an historical survey, supported by extracts of sources, of the origin of the word and the concept of « algorithm », as well as algorithms you can find in history of mathematics. Then, we'll analyze how the study of historical sources in the classroom, showing the algorithmic way of thinking of some mathematicians, allows students to get a better understanding of some mathematical concepts, for instance in the case of sequences, mainly the ones defined by recurrence relations.
We'll present didactical situations we have experienced with students (16-18 years old), built with extracts from Heron (*Metrica*), Bombelli (*Algebra*), and Fermat.
The sources will be given both in english and in french.

Renaud Chorlay (France) **Room A212**: *Making sense of the derivative, by combining historical sources and ICT.*
We would like to present three sets of historical texts, and three classroom activities based on them. Our goal is to help high school students make sense the new and thorny concept of the derivative. Needless to say the literature on this topic is huge: it shows quite clearly where the main difficulties lie, and offers many fruitful leads. We retained an approach emphasizing the importance of task solving, and designed a learning path which gradually expands the concept by unveiling new and

efficient aspects: local straightness, limit-position of secants, affine approximation, and iterated affine approximation. The use of original sources plays a part at several key stages.
The three sets of texts are as follows:

- Two ways to look at the tangent to the circle: in Euclid's *Elements* (definitions in Book III, and proposition III.16), and in Clairaut's *Elémens de Géométrie* (1st published in 1741), where we shall study Prop.18 of Part III.
- A Babylonian approximation rule for square roots. Source: D. Fowler & E. Robson, *Square Root Approximation in Old Babylonian Mathematics : YBC 7289 in Context*, Historia Mathematica n°25 (1998), p.366-375.
- Euler's presentation of Newton's methods for solving polynomial equation by successive approximations. Source: *Eléments d'algèbre, tome premier*, Lyon : chez Jean-Marie Bruyset, 1774. pp. 679 and fol.

Although based on historical sources, the three classroom activities do not use them similarly: in the first case, the analysis of proof-texts aims at triggering reflexion on definitions and proofs; in the second case, we considered the original Babylonian tablet was not fit for classroom use, and opted for the explicit use of one secondary source; in the third case, the text is actually used to learn a new and efficient technique.
In all three cases, heavy use is made of ICT: dynamic geometry software, spreadsheet, formal calculus software.
Clearly, our choice of original sources would have differed significantly, had we decided to teach the history of calculus rather than teach the derivative. In particular, we decided *not* to base classroom activities on some landmark texts in the history of mathematics; landmark texts which we *do* include in our teacher-training course on HM. We would like to end the workshop with a discussion on these two types of use of original sources, in terms of goal and criteria for texts selection.

Cecile de Hosson (France) **Room A214**: ***Using historical texts in an interdisciplinary perspective: two examples of the interrelation between mathematics and natural sciences.***

In this workshop two sets of historical texts will be studied with the aim of providing the participant with examples of effective interrelations between mathematics and natural sciences (physics and astronomy) according to the way these two scientific areas are defined today. These two sets of documents will refer to two different times : the Greek and Chinese Ancient World and the Italian Classical Time; they will include the following texts :
First set (Ancient Greek and Chinese cosmology)

- Ho Peng Yoke. (1966). *The Astronomical chapters of the Chin shu*. Paris: The Hague, Mouton & Co.
- Weir, J. (1931). The method of Eratosthenes. *The Journal of the Royal Astronomical Society of Canada,* 25, 294–297.

Second set (Galileo's theory of free fall)

- Galilei, G. (1638). *Mathematical discourses and demonstrations, relating to Two New Sciences*, trans. H. Crew and A. de Salvio (1914). Macmillan. (3rd day, th. II, prop. II).

In the historical texts forming the first set of documents two different cosmological models leaning on the same empirical observation are described and used. The participants will be introduced to the specificities of the experimental procedures (and the associated measuring instruments) and

their relationships with the mathematical tools involved. A pedagogical use of the two texts will be proposed with the aim of providing students with elements of understanding what a "model" is (from an epistemological point of view).
The second set of documents concerns Galileo's discovery of the law of free fall and its implication to inclined planes. They will be used in order to explore the manner in which purely mathematical considerations entered into Galileo's working on movement. Starting from Aristotelian's philosophy of movement, the participants will access Galileo's intellectual procedure where the conceptualization process is intrinsically connected to a mathematical processing.
The general objective of the workshop is to illustrate the intrinsic links connecting mathematics and natural science for the modelling process of the natural phenomena that forms an essential step for a rational comprehension of the world. Using historical grounds in the classroom could be a fruitful way of making students (and teachers) aware of aspects of the science enterprise (in terms of measuring, modelling, conceptualizing, etc.). In this regard, the math/science complex as revealed by some discovery processes has a promising part to play.

Susanne Spies, Gregor Nickel and Henrike Allmendinger (Germany) **Room A130**: *Using original sources in teachers education – An analysis on possible effects and experiences.*

The advantages of using original sources in school mathematics has been discussed widely (e.g. Jahnke et al. (2000), Jankvist and Iversen (2013)). In teachers education, history of mathematics is also attributed an important role, with a great variety of possible implementations (e.g. Arcavi and Isoda (2007), Beutelspacher et al. (2011), Nickel (2013)).
The integration of history of mathematics is one of the key features for the teachers' education at Siegen university; in addition to special courses on history of mathematics, we integrate historical aspects and work with original sources in didactics courses as well as in lectures on mathematics and elementary mathematics.
From a normative point of view, working with original sources can contribute to widely accepted aims of teachers' education, such as understanding the process character of mathematics and fostering authentic experiences in mathematical research on an elementary level. Furthermore, the alienation by historical mathematical sources can cause fertile confusions, that possibly enhance the perspective on current mathematical processes of conceptualizations as well as on student's and pupil's documents.
After discussing advantages and problems of an approach to original sources on an abstract level, we will present conrete examples and own experiences from courses on elementary mathematics as well as didactical seminars.

Tuesday July 15

<u>13:30-16:30: Workshops 3 hours: **Theme 1 & 7**</u>

Evelyne Barbin (France) **room A303**: *Curves in history and in teaching of mathematics: problems, meanings, classifications.*

In mathematical teaching nowadays curves appear as graphs of functions, there are more or less become an exercise of applications of calculus, and their place are less and less important. So the interest for curves disappeared for students and for teachers in secondary schools and also in

universities.
But in history of mathematics, from Greek geometry to mathematics of today, curves play an important role. History can learn many things to us. The first point is that curves are not only a pedagogical object to judge the competencies of students: they were invented to solve problems of geometry, optics, etc. Another point was to examine how the curves can be drawn, produced or constructed, and we find many possibilities given by mathematicians of the past. Last point but not the least was to classify the curves. The purpose of the workshop is to examine some historical steps in the long history of curves, and the goal is to reintroduce curves in teaching as a rich, interesting, open subject. We will examine the possibility to create an European Team of teachers and researchers working together to progress on this subject.
Texts taken in Geminus of Rhodes (1st century), the *Mathematical Collection* of Pappus of Alexandria (3^{d} century), the *Commentaries* of Eutocius of Escalon (6th century), the *Geometry* of Descartes (1637), papers of Van Schooten (1654) and Leibniz (1693), the *Introduction to the Infinitesimal Analysis* of Euler (1748), papers of Peaucellier (1868), and (Kempe (1877), the *Mechanisms for the Generation of curves* of Artobolevsky (1964).

Mikkel Willum Johansen and Tinne Hoff Kjeldsen (Denmark) **room A210**: ***Mathematics as a tool-driven practice: Workshop on the use of material and conceptual artifacts in mathematics.***

When we discuss how and to what extend new and powerful CAS-tools should be used in the mathematics education it is important to remember that mathematics is and always has been an essentially tool-driven activity. Both material tools (such as tallying sticks, drawing machines, various types of abacuses and various representational systems) and conceptual tools (such as rules, algorithms and concepts such as the number line) have been used for millennia. In general, such tools can be called cognitive tools or cognitive artefacts, as they are tools developed with the main purpose of assisting human cognition.

Studies of human infants and isolated tribes (such as the Amazonian Mundurukú and Pirañha people) have shown that humans have some inborn abilities to do mathematics, but also confirm that these abilities are extremely limited (see e.g. Nùñez 2009). We simply cannot do mathematics without the help of cognitive artefacts (and other types of cognitive technology). Thus, the range of mathematical problems we can take on and solve is clearly influenced by the cognitive tools we have at our disposal. Furthermore, it can be argued that the very content of mathematics, i.e. the objects and methods we accept as mathematical, also has been influenced and shaped by the development of new cognitive tools.

In this workshop we will introduce the participants to the concept of cognitive artefacts. On this basis, the participants will work with both some basic, historical examples of artefact-use in mathematics, and a more recent example, where an artefact played a vital part in the development of a mathematical theory. It is not necessary for the participants to prepare in advance for this workshop. We will provide the texts and the materials, and the participants will work on these as part of the workshop guided by us. On this basis we will discuss a) mathematics as a tool-driven activity and b) to what extend CAS-tools is an extension of the existing practice or something new.

Material
In the workshop we will use material that is condensed and rewritten from (Johansen, 2010),

(Kjeldsen, 2009), and extracts of sources from H. Minkowski's collected works (Minkowski, 1911).
Literature

- Johansen, M. W. (2010): *Naturalism in the Philosophy of Mathematics*, (Copenhagen: Faculty of Science, University of Copenhagen). (Online publication available at: http://www.nbi.dk/natphil/prs/mwj/Dissertation-mwj2010.pdf).
- Kjeldsen, T. H. (2009): Egg-forms and Measure Bodies: Different Mathematical Practices in the Early History of the Development of the Modern Theory of Convexity. *Science in Context*, vol. 22 (1), pp. 85-113.
- Minkowski, H. (1911). Über Geometrie der Zahlen. *Gesammelte Abhandlungen*, vol. I. Leipzig, Berlin: B. G. Teubner.

Núñez, R. (2009): "Numbers and Arithmetic: Neither Hardwired Nor Out There", *Biological Theory* 4(1).

Francois Plantade (France) **room A212**: ***Jules Houël (1823-86): a French mathematician well connected to the Nordic mathematicians in the second part of the XIXth century***.

The French mathematician and astronom Jules Houël (1823-86) taught especially real and complex analysis in the Bordeaux Science Faculty from 1859 until 1884. He was known as polyglott and a brilliant computer ; he diffused the ideas of noneuclidean geometries into France. He founded and co-chaired – from 1870 until 1883, with G. Darboux - the "Bulletin des sciences mathématiques et astronomiques", which should inform French scientifics of European mathematics and astronomy. So Houël was connected to many European mathematicians. The study of the Houël's remaining correspondences shows two geographic poles : Italy (Battaglini, Bellavitis, Beltrami, d'Ovidio, Forti, ...) and Scandinavy (C.A. Bjerknes, Dillner, Lie, Mittag-Leffler, Zeuthen). With Italian mathematicians, Houël discussed especially noneuclidean geometries and their diffusion and with Nordic ones complex analysis, elliptic functions and also Abel's work and life. The second point will be the object of the present workshop. The most Houël's important Nordic correspondent – in term of durée and quantity – was J. Diller, assistant professor in Uppsala University and editor of the Journal "Tidskrift för mathematik och physik". But none a letter between both has been found ; we only know things about by other correspondents. Dillner is an important person as the "supervisor" of Mittag-Leffler's Ph.D and encouraged him to write to Houël.. The correspondence between Mittag-Leffler and Houël lasted from July 1872 until may 1883. It is really interesting because Houël is the first non scandinavian mathematician whom Mittag-Leffler wrote to so we can follow the genesis of his ideas and the many issues are covered. Obviously, the start pointing is mathematics and more precisely complex analysis. Repeatedly, Mittag-Leffler and Houël discussed of the theory of functions of one complex variable and elliptic functions ; they discussed also the ways of teaching them. The organization of mathematics education in Europe and especially in France and Germany is a recurrent topic. Finally, the mathematics journals are omnipresent in the correspondence. Houël corresponded with S. Lie, a collaborator ; in the early 80s, Lie informed him of the publication of the N.H. Abel's biography in Norwegian by C.A. Bjerknes ; the old Houël decided to translate into French in order to diffuse the mathematical ideas and the the life of that great genius. Houël asked Bjerknes help for translating it. They corresponded from beginning 1882 until may 1885 ; after may 1885, Houël could not work anymore : he died in June 1886. So that translation is the last work of Houël . Houël asked his name not to be written on the publication : he found his work not enough successful... We will provide letters translated in English from Houël to Mittag-Leffler, to Lie, to Bjerknes and from Mittag-Leffler, Bjerknes to Houël in order to present those strong connections between Houël and Nordic mathematicians.

Leo Rogers (UK) **room A214**: ***Tools and Procedures for Using Historical Materials in the Classroom****.*

The workshop will be based on ideas in my paper (Rogers 2011) where I will develop the principles of using concept maps of 'significant' items pertaining to the *history* of mathematics and building a narrative of relevant *heritage* content (Grattan-Guinness 2004) from where we can develop particular orientations relevant to specific classroom contexts. (See Map, Narrative and Orientation in 2011: 7-13 and Appendices)
Some examples will be presented from workshops used with teachers and secondary pupils (ages 11 – 18) where problems adapted from historical contexts will be offered for criticism to participants. The main objective of the workshop is to discuss the manner in which these problems or others like them may be introduced in the classroom *to foster the pupil's own epistemological process* in building up their own mathematical knowledge.
Colleagues attending this workshop who have used historical material with students are invited to bring their own examples of classroom problems for discussion.
Some questions to consider about affordances and limitations when using historical materials as classroom problems:

- Can this material be used (or adapted) with pupils at any age
- What mathematics (if any) do pupils need to know in order to address the problem
- What kinds of problem-situations is this material designed to raise
- What is its potential for developing conceptual knowledge
- Does it have relevance for building a knowledge of mathematics as a science
- Do the ideas involved appear in different areas and at different levels of mathematics
- Does this material encourage mathematical communication
- Does this material encourage teachers' own reflection processes

Some References:
Grattan-Guinness, I. (2004) The mathematics of the past: distinguishing its history from our heritage. *Historia Mathematica.* **31** (2) 2004 (163–185)
Rogers. L. (2011) Mapping our heritage to the curriculum: historical and pedagogical strategies for the professional development of teachers. In Katz, V & Tzanakis, C. (2011) *Recent Developments on Introducing a Historical Dimension in Mathematics Education.* Washington. MAA.
Watson, A. Jones, K. and Pratt, D. (2013) *Key Ideas in Teaching Mathematics: Research-based guidance for ages 9-19.* Oxford. OUP

Desirée Kröger and Sara Confalonieri (Germany) **room A303**: ***Learning mathematics from German and French textbooks in the 18th century - the case study of negative numbers.***

In the 18th century the negative numbers were accepted and used, but not rigorously justified until the 19th century. For that reason it is interesting to take a look at different approaches to the negative numbers. As sources, we choose a few passages from some German and French textbooks.

After a brief presentation of the French and German circumstances (educational system, institutional conditions, position of mathematics, textbooks and their authors,…) the participants should work in teams on the different sources. There should be teams in order to work on the German, French, and English sources, and also one team that can present today's approaches. The aim is to show up the differences among the various contemporary at that time and the developments that lead to today's approaches of negative numbers. We propose the following questions:

- Placement: Where do we find the negative numbers in the textbooks?
- Definition: Is there a definition of the negative numbers?
- Terminology: Which expressions are used?
- Are the explanations clear and vivid? Are there any examples?
- Are there any historical or philosophical remarks?
- Is the difference of plus and minus once as arithmetic operators, once as algebraic signs clear?
- Are there any parallels or differences to today's approaches?
- Are there any models for negative numbers?
- Calculating with negative numbers?

References:

BÉLIDOR, Bernard Forest de: *Nouveau cours de mathématiques à l'usage de l'Artillerie et du Génie*, Paris, 1725.

BÉZOUT, Étienne: *Cours de mathématiques*, Paris, 1764-1769.

CAMUS, Charles: *Cours de mathématiques*, Paris, 1749-1751.

EULER, Leonard: *Elements of Algebra*, London, 1822, pp. 3-6, 6-10, 13-15.

KÄSTNER, Abraham Gotthelf: *Anfangsgründe der Arithmetik, Geometrie, ebenen und sphärischen Trigonometrie und Perspectiv (= Der mathematische Anfangsgründe ersten Theils erste Abtheilung)*, Göttingen, Vandenhoeck und Ruprecht, ^{6}1800, pp. 71-80.

WOLFF, Christian: *Anfangsgründe aller mathematischen Wissenschaften*, vol. 4, Halle, Renger, 1775, pp. 1549-1562.

Thursday July 17

14:30-17:30: Workshops 3 hours: **Theme 1, 4 & 6**

Xavier Lefort (France) **room A210**: ***Quelques etapes historiques du reperage en haute mer aspects mathematiques / Some stages of the location in open sea***

Se situer en haute mer, loin des côtes, a toujours été le principal problème de la navigation hauturière. Les seuls repères possibles se trouvent dans le ciel, soleil, lune et étoiles. L'utilisation d'un système géocentrique a perduré jusqu'à aujourd'hui, conduisant à d'intéressants calculs en géométrie sphérique.

L'atelier proposerait un historique des méthodes de repérage, en s'arrêtant à quatre étapes : Pedro de la Medina (L'Art de naviguer 1569), Simon Stevin (De l'histiodromie 1634), Etienne Bézout (cours de navigation 1781), Edmond Dubois (Ecole Navale : Cours de navigation 1869). Lire et refaire les calculs issus des ouvrages de ces auteurs pour retrouver leurs résultats et les commenter pourraient faire ressentir aux participants les sensations du grand large et … les mouvements du navire !

Some stages of the location in open sea

Being situated in open sea, far from coast, has always been the main problem of the open sea navigation. The only possible marks are in the sky, the sun, the moon and the stars. The use of a geocentric system has been going on until today, leading to interesting calculations in spherical geometry.

The workshop would propose a history of the methods of location, with four stages : Pedro de la Medina (Ars de navigar 1569), Simon Stevin (The Histiodromie 1634), Etienne Bézout (Cours de navigation 1781), Edmond Dubois (Course de navigation à l'Ecole Navale 1869). Reading and doing again the stemming calculations works of these authors to find their results and comment on them could make the participants feel the sensations of the open sea and … the movements of the ship !

Carole Nahum (France) **room A212**: *A Strong Collaboration between Physicians and Mathematicians through the XIXth Century: Double Refraction Theory*.

Mathematics and Physics may address the same topics though they employ different methods, experiments on one hand, analysis or geometry on the other. Nevertheless, those two disciplines stimulate one another which usually generate progress.

In order to illustrate this fact, we propose to draw some examples from Augustin Fresnel's wave theory of light, elaborated in 1819.

In 1830, famous mathematician Augustin-Louis Cauchy forecasted elliptical polarization by crystals (phase jump) trying to transcript analytically Fresnel's principles. This phenomenon had not been considered earlier. It was checked by Physicist Jules Jamin, about twenty years later.

Also, in 1832, some mathematical calculations led William Rowan Hamilton to discover conical refraction. A year later, Humphrey Lloyd observed this phenomenon.

However, we shall present an even more surprising interaction between the two sciences, which emerged from crystallography. A whole community of researchers has been animated by the problem of determining the properties of wave surfaces, since the directions of light through crystals can be deduced by Huygens' geometrical construction. Mathematicians invented new tools which led to now famous theorems.

History of Sciences shows the importance to teach physics and mathematics coherently rather than as two separated independent fields. Also researchers from each discipline would take advantage in adapting their language when communicating their discoveries.

Abstracts from Christian Huygens' Treatise on Light 1690, Augustin Fresnel's Memoir on double refraction 1821, William Rowan Hamilton's Memoir on Systems of Rays 1830, Archibald Smith's short paper on Wave Surfaces, will be read and commented.

Thomas Preveraud (France) **room 214**: *Geometry, Teaching and Institutions in 19th Century United States. A Study of Legendre's Geometry Translations Corpus.*

In the beginning of the 19th century, the practice and the diffusion of mathematics within the United States was unprecedentedly transformed. The question of the teaching of geometry in the colleges that used to lean during the 18th century only on British versions of Euclid's *Elements* - like *Elements of Geometry* from Scottish mathematician John Playfair - strengthened as some of colleges curriculums were reformed. For many teachers and educationalists, teaching of geometry had to fit two requirements: in one hand, training learned minds to rhetoric and deductive reasoning, in the other hand, avoiding useless and time consuming speculations.

In the twenties, *Elements of Geometry*, by Adrien-Marie Legendre, published in France in 1794, seemed to match these expectations since it was first translated at Harvard by John Farrar and then at West Point Military Academy by Charles Davies. The two translations were spread and used in several universities courses during the first half of the 19th century.

After 1850, plus these two textbooks reprints, turning themselves to secondary teaching, three new translations of *Legendre's Geometry* were designed in the United States in three different educational contexts: Elias Loomis (1849) for civil high education, Francis H. Smith (1867) for Virginia Military Institute and James Thomson (1847) for high schools.

This workshop aims to analyze the 19th United States *Legendre's Geometry* translations corpus, looking at the comparison and the evolution of the textbooks.

First, the workshop will show the place of these translations in the editorial background of the teaching of geometry in the United States. It will afterwards analyze the evolution of contents regarding several items: algebrization, *reductio ad absurdum*, structure of the books, role of figures and problems. It will examine the publication contexts (high schools, universities, military schools) and will give examples of the influence of the targeted readership upon the writing and the adaptation in the translation process. Last, the workshop will show how the editorial sliding of the translations reprints published before 1850 and of the new productions published in the second half of the 19th century answered to the evolution of the place and function of geometry in American instruction.

The texts that will be studied during the workshop will be:
Charles DAVIES, *Elements of Geometry and Trigonometry*, New York, Ryan, 1828.
John FARRAR, *Elements of Geometry by. A.M. Legendre*, Cambridge, Hilliard and Metcalf, 1819.
Elias LOOMIS, *Elements of Geometry and conic sections*, New York, Harper & Brothers, 1849.
Francis H. SMITH, *Elements of Geometry by A.M. Legendre,* Baltimore, Kelly & Piet, 1867.
James THOMSON, *Elements of Geometry on the basis of Dr. Brewster's Legendre,* New Haven, Durrie and Peck, 1844.

Gert Schubring (Germany) **room A130**: ***Workshop on new approaches and results of research into the history of mathematics education.***

This workshop will be a sequel to the invited lecture to topic 6: New Approaches and Results in the History of Teaching and Learning Mathematics. We will study various texts, which enable to deepen the questions and issues raised in the plenary lecture. In particular, texts will be used from different methodological approaches, allowing thus to obtain a comparative understanding.
Special emphasis will be given to two issues:

- differences between various countries regarding the processes by which mathematics achieved the status of a major teaching discipline in secondary schools,

- socio-political dimensions in the development and realization of *Mathematics for All*.

Caroline Kuhn and Snezana Lawrence (UK) **room A104**: ***Personalised Learning Environment and the History of Mathematics in the Learning of Mathematics.***

The proposed talk will describe the project, which aims to design and implement a personalized learning environment built around contextual historical material for the learning of mathematics. On the one hand, the project seeks to understand the principles that connect the personalized learning and digital technology, both as ways of providing individual input and collaborative learning at a distance; on the other hand it seeks to examine the role that history of mathematics may have in such a learning environment.

The talk will therefore concentrate on three aspects:

1. It will survey the existing and historical examples of personalized learning environments which use the history of mathematics as a contextual tool for the learning of mathematics

2. It will question the hows and whys on using the history of mathematics to underpin the epistemological aspect of mathematics education in digital environments

3. It will question whether the original sources, widely available on the Internet, can contribute to creating an authentic personalized learning environment, which rests on original research in mathematics.

The talk will be illustrated by the examples of personalized learning environments in mathematics that use some aspects of the history of mathematics already existing in the digital world. It will attempt to propose a brief for creating a personalized learning environment, which has at its core the historical context of the development of mathematical sciences. Whilst the project is a recent collaboration between two authors, and empirical studies of the students' preferences in the learning of mathematics in digital environments is not abundant, we will aim to produce results of our initial data.

Frédéric Metin and Patrick Guyot (France) **room A220**: ***What can we learn from Jacques Rohault's lessons in mathematics and physics?***

Jacques Rohault (1618 - 1672) was often referred to as 'the famous Monsieur Rohault' in contemporary English books about natural philosophy. He was regarded as the most important Cartesian philosopher, and his posthumous works were published several times in London, in Latin as well as in English, even after his own homeland had forgotten him.

Rohault was also a famous teacher: his public lessons in Paris were attended by a variety of persons, as Malebranche witnesses. The lessons on physics were especially appreciated, for they were spectacular and based on experimentation; but we must not forget that Rohault was also a teacher of mathematics for the Dauphin of France, and as such he taught Euclid's Elements (the first six books), trigonometry, practical geometry, fortifications etc. The course for the Dauphin has been taught in other private lessons, the manuscripts of which still exist in public libraries.

The reading of Rohault's writings, pointing out the differences between printed and manuscript versions, offers a view on a great mind of his century, inventor of pedagogical choices based on the personal implication of the one who teaches, on debate and controversy.

Suggested texts (to be studied during the workshop; the text in French will be “made English”):

- Œuvres posthumes de Jacques Rohault. Paris, 1680.
- Course on trigonometry and fortification (manuscripts), ca 1670.
- Rohault’s System of Natural Philosphy, illustrated with Dr Samuel Clarke’s notes, taken mostly out of Sir Isaac Newton’s Philosophy. London, 1723.