Abstracts for oral presentations

Monday July 14

<u>16:30-17:30: Oral presentation: **Theme 2 & 3**</u>

Room A210:
Panagiotis Delikanlis (Greece): ***Meno by Plato: an ancient inexhaustible mine of knowledge***.

Meno by Plato is a middle period dialogue. It was written about 385-386 B.C. The dialogue begins with Meno asking Socrates whether virtue can be taught. Socrates states that he does not know the definition of virtue. Meno is in *aporia* (puzzlement) and responds with o paradox. Socrates introduces the recollection as a theory of knowledge (Since, it would seem, research and learning are wholly recollection, Plat. Meno 81d). Afterwards, he illustrates his theory by posing a geometrical problem: "A square of side two feet has area four square feet. Doubling the area, we draw another square of eight square feet. How long is the side of the new square?".

In this oral presentation I would like to discuss:

- Solutions using different types of square paper
- Socratic method solutions
- Pick's formula solution

Square paper is a mediating tool. Tools are mediators of human thought and behaviour. The use of square paper supported ingenious geometry solutions.

Fanglin Tian (China): ***Integrating the History into the Teaching of the Concept of Logarithms.***

There are lots of theoretical discussion on the role of history in mathematics education and the relation between history and mathematics education. It seems that we lack the empirical studies which could link the history with mathematics education. In this lecture, we would like to report a case study in which the history of mathematics is integrated into a normal class in China.
The mathematical concept of logarithms plays a crucial role in many aspects of human existence. In the traditional curriculum, logarithms are introduced as exponents, just as Euler's definition of logarithms. However, historically, the invention of logarithms was completely independent of exponents. That's a miracle in the history of mathematics and the fact has been ignored in our teaching before. As many mathematics teachers in senior high schools said, students are always confused with why they have to learn the concept of logarithms and what the meaning of the sign of "log" is, and they are inclined to forget the logarithmic properties.
There are four approaches to integrating the history into mathematics teaching. They are complementation approach, replication approach, accommodation approach and reconstruction approach. In this lecture, we would like to present how we, a team of researchers, cooperated with a team of experienced senior high school teachers, to develop the design of the lesson, how to cite the story about Napier, and how the four approaches mentioned above are used in the first lesson

about the concept of logarithms to help students experience the necessity of the invention, deepen their understanding of the concept and remind them to remember and appreciate the efforts and contributions of past mathematicians and learn from them.
What's more, we'd like to show the results of questionnaires and interviews on students, we could easily see that the goals of knowledge and affection are both attained. Though there are some aspects needed to improve, the results are generally satisfactory.
Besides students, the teacher also harvests a lot during the process. It's quite helpful for the teacher's professional development. Some of the her changes with knowledge and attitude are presented in this lecture. In fact, there are lots of excellent teachers in China, we would like to share some ideas about the use of history in Mathematics education from some experienced Chinese mathematics teachers.

Room A212:

Morten Misfeldt, Kristian Danielsen and Henrik Kragh Sørensen (Denmark): ***Cognitive conflicts and exploratory experimentation: How can the continuous overlap between empirical and deductive proof schemes be conceptualized?***

Background
The tendency among upper secondary students to "prove" mathematical statements by examples rather than by universal deductive reasoning is an established research finding in mathematics education research (Arzarello et al., 2011). This educational problem is described as students having "empirical proof schemes" opposed to "deductive proof schemes". Phrased in these terms, empirical studies have shown that students have difficulties performing and internalizing the movement towards deductive proof schemes and that empirical proof schemes even stand in cognitive conflict with the acquisition and performance of deductive proofs.
Research question
In order to achieve a deeper understanding of the nature of students' problems in making the transition to more formal and deductive proof, we propose an intervention project that scaffolds the students' movement from open explorative activities to the development of proof sketches and ideas. In the process we investigate whether and how the students' conception of proofs and explanations in mathematics change.
We work with the hypothesis that the problem with supporting students' transition to deductive proofs can at least partly be explained as a problem of bringing their empirical investigations into the deductive proof process in relevant and productive ways. Thus, we want to argue for a continuous overlap between the proof strategies that should also manifest itself in the proof schemes.
Intervention and method
We are developing materials that enable students to work in empirical situations with lattice polygons. The material emphasizes (computer mediated) student experiments and scaffolds their development of proof sketches based on their experiments. The material is developed as a prototype of an epistemic game that involves cycles of action, experimentation/investigation and reflexion.

Through the analyses of the students' portfolios and deliberations, we are able to assess the students' performance of proofs and the conceptions of mathematical methodology before and after the intervention (as analysed through questionnaires).
At the summer university we will be able to present data from the intervention.
References

Arzarello, F., Dreyfus, T., Gueudet, G., Hoyles, C., Krainer, K., Niss, M., … Vershaffel, L. (2011). Do Theorems Admit Exceptions? Solid Findings in Mathematics Education on Empirical Proof Schemes. *EMS newsletter*, (December), 50–53. Retrieved from http://www.euro-math-soc.eu/ems_education/Solid_Findings_Proof_Newsletter.pdf

Hong Yanjun (China): ***Teaching the Area of a Circle from the Perspective of HPM.***

In China, importance is attached to three objectives in mathematics teaching: knowledge & skills, process & methods, affect & attitude, corresponding to which we have the following "whys" of integrating the history of mathematics into mathematical teaching:

(1) The history of mathematics is helpful for deepening students' understanding of mathematics;

(2) The history of mathematics provides a lot of problem-solving methods and can broaden students' thinking;

(3) The history of mathematics increase students' interest and create their motivation.

Can the above values be really achieved in the mathematics classroom? To answer this question, the history of mathematics was integrated into the teaching of the area of a circle at the sixth grade in a junior high school in Shanghai. Based on the rules of interest, authenticity, effectiveness, acceptability and innovation, Kepler's method of finding the area of circle, which was realized by means of the Geometer's Sketchpad, was adopted in the instructive design. Through classroom observation, questionnaire survey and interviews, it was found that most students were fond of the ways of teaching integrated with the history of mathematics, deeply impressed by the story about Kepler and his approach to the area of the circle and that they understand the cut-and-complement method well. The following benefits of the history of mathematics in the classroom teaching are identified from the students' viewpoints:

(1) The history of mathematics is very interesting and can attract the students' attention;

(2) The methods from the history of mathematics which are restructured by teacher are easy to understand for students;

(3) The history of mathematics can extend students' knowledge;

(4) The history of mathematics tell students that there are variety of methods of solving one and the same problem.

Therefore, all the three objectives of mathematics teaching can be achieved when the history of mathematics are suitably used.

Room A214:

Zou Jiachen (China): ***The Genetic Approach to Teaching the Ellipse.***

In this paper, we study the history of conic sections. By dating back to its history, we find out the

relationship between the origin of the ellipse and the definition in today's math textbooks, and also a simple method to derive the equation of the ellipse. Meanwhile, we make a comparative study on ways of introducing the ellipse used in 14 different math textbooks in history. Then, we reconstruct the history of the ellipse according to the idea of genetic approach to teaching mathematics. Based on the reconstruction of history, we design three different ways to teach the ellipse and compare these ways with the two ways based on textbooks in China. What's more, one of the three teaching designs was put into practice and achieved big success.
A questionnaire was designed to grasp students' intuitive understanding of the ellipse. The purpose of this survey was to know about students' understanding of the first definition of the ellipse and the way in which textbooks used to introduce the ellipse, to grasp students' understanding of the proposition that one of the sections between a cylinder (or a cone) and a plane creates an ellipse and to know about students' acceptance of the way of introducing the ellipse using the Dandelin spheres. We also investigated both teachers' and students' selection of teaching methods used in the introduction of the ellipse definition and its equation derivation. The questionnaire survey was conducted to 723 high school students and 40 high school mathematics teachers in Shanghai, China; an interview was also conducted to seven teachers after the questionnaire survey. By analyzing the survey and interview materials, we got the following conclusions.
(a) Historical parallelism exists when it comes to students' intuitive understanding of the ellipse. Students usually understand the ellipse as "a squashed circle". The majority of students understand the first definition of the ellipse, but they are confused about the way to introduce the ellipse in textbooks.
(b) Students can understand that one of the sections between a cylinder (or a cone) and a plane creates an ellipse, and the 3-dimensional model using the Dandelin spheres. Students make no difference on understanding these two points whether they have learned solid geometry. That is to say, students accept the teaching design based on the Dandelin spheres without difficulties. So it is possible to use this method in classroom teaching.
(c) There is an obvious difference on the choice of how to introduce the definition and derive the standard equation of the ellipse between teachers and students. Students prefer to the genetic teaching, while teachers prefer to the ways based on textbooks.

Michel Kourkoulos and Costas Tzanakis (Greece): ***Statistics and free will.***

In his published works from 1829 till 1869, Quetelet has shown that events like crimes, suicides and marriages present a remarkable statistical stability in the course of time, on the condition that social circumstances in a given country or State remained approximately unchanged. This stability allowed an accurate anticipation of statistical results for the years to come, on the condition of social stability as well.
On the other hand, in such events, free will plays an important role. According to conceptions at that time, events that imply the existence of human free will should escape any possibility of prediction.
Quetelet interpreted these statistical results by arguing that when a large number of such events is examined, the influence of the free will of human individuals disappear, since the influence of the free will of some individuals compensate that of others and that variations of free will among individuals cannot yield important variations in the statistics of the above mentioned events, of course always supposing the approximate stability of social conditions.

At that time, these statistical results and Quetelet's interpretation stimulated the debate on human free will and its restrictions. In the context of this debate, Quetelet was criticised for being promoting fatalism and materialism. He worked to refute this criticism by elaborating his argumentation further and providing new statistical results to support it.

In an introductory seminar on Statistics for prospective primary school teachers[1], we presented Quetelet's statistical data and arguments, in order to initiate a discussion on the subject of free will. Our motivation was to examine in detail a possible example of raising philosophical issues and their discussion on the basis of specific (in this case) statistical questions and data, thus pointing out the possibility to interrelate statistics and more generally, mathematics teaching, with philosophy, the emphasis being on outlining and signalling a possible path from mathematics to philosophy.

During the discussion of Quetelet's arguments and data, students identified restrictions on the application and influence of free will, as well as, external factors that may influence free will.

As they identified such restrictions and factors more and more, several questions emerged like: Does human free will really exist? If yes, to what extent human will is really free? What is the domain where this is applicable? In the follow-up discussion, we have introduced additional information on Quetelet's argumentation and data and brought in the argumentation on this subject of other scholars of that period or of our time (like Laplace, Kant, Kane, Nagel). Students also surveyed part of the existing literature on the subject and brought in the argumentation of other scholars on this subject which they found interesting and fascinating.

During this part of the discussion, students expressed a whole spectrum of different arguments and points of view: From the point of view that there is very little space for human free will, up to the point of view that human free will does exist and there is enough space for it to be unfolded, playing a crucial role in one's own life and character development.

At the end of the discussion, students had a better idea of the great extent and depth of the debate on free will and the diversity of the positions held by different thinkers in the last two centuries. In addition, most of the students thought that their own opinions were provisional or preliminary, susceptible to change upon more profound reflection, though some others considered their opinions to be more firm[2]. All of them were helped to get a deeper awareness of what statistics really is and appreciate that even a simple mathematical treatment of given empirical data may be related to raising and discussing philosophical issues that are important for any human individual. This made a strong impression on all students and there are indications that their affective predisposition to statistics, and more generally, to mathematics was positively influenced. Using historically important original texts was a key factor that permitted the stimulation of students' interest and deepened their awareness of what statistics is about. More precisely, it gave them the opportunity to come into contact with the philosophical debate raised by specific mathematical developments (namely, Quetelet's effort to develop "moral statistics" and in particular his work concerning statistics and free will) and thus go through other philosophers and scientists' work pertinent to this debate. We believe that it would be much more difficult to elaborate on this subject and show the relevance of mathematical results and thinking, without appropriate original sources like those we have used.

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[1] The seminar was attended by 30 3rd and 4th year undergraduate students of the Department of Education of the University of Crete.

[2] For these students, this seems to be related to firm preexisting convictions.

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Room A130:

Maria Zoraide Martins Costa Soares, Rosa Maria Machado and Otília Terezinha Wiermann Paques and Douglas Daniel (Brazil): ***Solving some Lamé's problems using geometric software***:

The mathematics teaching laboratory (LEM/UNICAMP), Brazil, currently has as aim to research the use of history of mathematics in teaching, using geometry software. This paper is concerning about two problems of the French mathematician Gabriel Lamé (1795-1870) with solutions using geometry software. We can find these problems in the book *Examen des diferentes methodes employées pour résoudre les problèmes de géometrie* (Paris-1818).

The view of Lamé for the problems was only algebraic. With the geometry software, we have dynamic solutions that completes the Lamé's solution and brings us a global analysis by the movement. These new view of the problem take us to add real conditions for the Lamé's solutions.

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Room A104:

Mario Sanchez Aguilar (Mexico): ***A didactic proposal for using the difference between two quantities to analyze the relationships between some magnitudes of the circle.***

The latest reform of secondary education in Mexico (Secretaría de Educación Pública, 2006) indicates that pupils should study geometric shapes and their properties, and the use of dynamic geometry software is recommended to support the study process. However, the teaching guides of this reform often indicate what topics should be taught but they don't clarify or suggest how they might be taught. For example, the written curriculum indicates that the relationship between the length of the radius and the area and circumference of a circle should be analyzed, but it doesn't provide enough details on how to analyze such relationship, nor how the teacher could use the dynamic geometry software to support the analysis.
In this work a didactic proposal aimed at secondary mathematics teachers is presented. Its purpose is to analyze the relationship between the length of the radius and the area and circumference of a circle, using the dynamic geometry software GeoGebra. The didactic proposal is inspired by a interpolation method called *methodus differentialis*, which was first used by Isaac Newton as a tool for predicting the behavior of some celestial bodies (Newton, 1686). In particular, we use the idea of the difference $x_2 - x_1$ between two consecutive quantities x_1 and x_2, which is the basic mathematical notion behind the *methodus differentialis*.
It's still necessary to test the didactic proposal in regular mathematics classrooms to learn more about its scope and limitations.

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Tuesday July 15

16:30-18:00: Oral presentation: **Theme 1**

Room A303:

Michael N. Fried (Israel) and Hans Niels Jahnke (Germany): ***Toeplitz's 1926 Paper on the Genetic Method: Meaning and Context.***

In 1926, Otto Topelitz delivered an address at a meeting of the German Mathematical Society held in Düsseldorf. It was published the following year in the *Jahresbericht der Deutschen Mathematiker,* as "The problem of university infinitesimal calculus courses and their demarcation from infinitesimal calculus in high schools." Toeplitz's purpose is to set out the difficulties encountered in creating an introductory course on the calculus for university students. Many of the difficulties which were Toeplitz's concern are still with us. He discusses, for example, the difficulty of balancing rigor with intuitiveness, of maintaining interest while not sacrificing depth and of maintaining depth without losing a sense of the whole; he discusses the difficulty of assuring that students' acquire tools without becoming mere technicians with no real taste for the calculus, with no sense of its beauty; and he discusses the problem of answering the needs of different kinds of students. Toeplitz recognizes these involve true dilemmas, but he sees a way out of them by way the history of mathematics. It is in this paper then that he delineates his idea of the “genetic method,” later exemplified in the posthumously published book *Die Entwicklung der Infinitesimalrechnung: Eine Einleitung in die Infinitesimalrechnung nach der genetischenMethode, vol.1* (1949) or in its English version, *The Calculus: The Genetic Approach*. (1963).

The “genetic approach” still is very suggestive to mathematics educators interested in the use of history of mathematics in mathematics education, since it supplies a rationale for studying history that does not trivialize history of mathematics; it shows how history of mathematics can not only supply content for mathematics teaching but also, as Toeplitz is at pains to emphasize, a guide for examining pedagogical problems. At the same time, as we shall discuss in our presentation, an attentive reading of Toeplitz’s 1926 address brings out tensions and assumptions about mathematics, history of mathematics and historiography. It is this that makes it of interest not only for educators, but also for historians of mathematics. In general, we hope our listeners with leave with a sense of how, even without an immediate intention to use history in the classroom, the history and historiography of mathematics and the teaching of mathematics may still be deeply entangled.

Ladislav Kvasz (Czech Republic): ***Degrees of inconsistency***.

Several mathematical theories, as for instance Leibniz’s differential and integral calculus, Newton’s theory of fluxions and fluents, Euler’s infinitesimal calculus, Frege’s foundations of arithmetic, Dedekind’s variant of set theory, or Peano’s theory of natural numbers -- they all were first formulated in a logically inconsistent form. Only after a certain period of time consistent formulations of these theories were found—by Cauchy, by Russell or by Hilbert respectively. The paper analyzes several historical cases of this “initial inconsistency”. It suggests distinguishing three kinds of inconsistency according to the “distance” of the proposed inconsistent theory from its consistent formulation. In teaching we usually ignore the inconsistent theories and we start the teaching of the calculus, of mathematical logic or of set theory straight with the consistent theories. The main thesis of the paper is that inconsistencies of different degree have different cognitive role and thus could be given different place in mathematics education.

Alain Bernard (France) and Katalin Gosztonyi (Hungary): ***Series of problems: a genre at the crossroad of various traditions and the possible point of departure of teachers’ reflections***

The presentation is focused on an interdisciplinary research project initiated three years ago (2011) in the framework of the labex HASTEC, an "excellence cluster" of various research laboratories dealing with history and anthropology of religion, science, philosophy and technologies. The project in question is focused on a generic form of historical source that we have called series of problems, namely any text or document constituted by chains of questions, problems or enigmas, followed by some form of answers (extended presentation, in French, here: http://problemata.hypotheses.org/28). Among these series of problems are series of mathematical problems, but one of the interest of the project is that the latter are only part of the genre of series of problems as a whole. The purpose of the project is to pursue, on a comparative basis, the study of such series of problems by making the fundamental hypothesis that they stand at the crossroad of various "cultures", the term referring not only to 'anthropological' cultures (the most usual sense of the word) but also to professional cultures (trading, teaching, law-making, etc.) or intellectual cultures (commentary, contests, encyclopedic projects..).

In 2012 and 2013 the project has been associated to a three-days training session directed toward a mixed audience of in service teachers of mathematics, history and literature, and students engaged in a master or doctorate degree in history or anthropology (presentation in French here: http://problemata.hypotheses.org/172). Our current project for 2014-15 is to "extend" this training session with a group of reflection associating researchers involved in the project and teachers or students interested in deepening the issues raised within our workshops, including the training sessions. The basic purpose of this prospective communication is (a) to explore schematically the possible aims and modalities of such a working group and (b) to check what international collaborations could made possible by it. The basic -and by no way trivial- question on which this collective endeavour should be focused is whether the study of series of problems has an interest for improving teaching in various disciplines and which one exactly. We shall explore the several ways in which this basic purpose might be understood.

Room A210:

David Guillemette (Canada): ***Sociocultural approaches in mathematics education for the investigation of the potential of history of mathematics with pre-service secondary school teachers.***

Since the mid 90s, reorientation (*dépaysement épistémologique*) have been a recurring concept in literature concerning the use of history in mathematics education (Barbin, 1997; 2006, Jahnke & *al*, 2000). Articulated mainly with the use of primary sources and the teachers' training context (Furinghetti, 2007; Lawrence, 2008), the assumption of reorientation emphasizes that history of mathematics touch and astonish pre-service teachers by the diversity of mathematics across different historical, cultural and social contexts. In contact with its history, future teachers raise many reflections on the nature, forms and uses of mathematical objects, which can lead to a cultural understanding of mathematics by inviting to historical-anthropological reflections on mathematics ; a repositioning of the discipline as a "human" activity (Barbin, 2012). The mention of this phenomenon in many papers from the last congress of the *International Study Group on the relations between the History and Pedagogy of Mathematics* (HPM 2012 (Wang & Choi, 2012)) shows how discussions and tensions on the subject, especially epistemological, are still very lively in the field. However, few empirical studies seem to have investigated this hypothesis directly. Difficulties, particularly methodological ones, seem to restrain conceptual developments around this major argument and the development of adapted training environments (Guillemette, 2011). My doctoral research project's goal is to describe the reorientation experienced by a group of six pre-service secondary teachers taking part of a history of mathematics courses. Seven activities concerning the reading of historical texts (A'hmosè, Euclid, Archimedes, Al-Khwarizmi, Chuquet, Roberval, Fermat) were constructed and implemented. Both synchronic and diachronic readings were performed (Fried, 2007; 2008). Data were collected during video recordings of these activities, individual interviews and a group interview.
As a first step, I will highlight how the development of a conceptual framework supported by sociocultural approaches in mathematics education (Radford, 2011; 2013, Roth & Radford, 2011), as well as a methodological framework articulated with a dialogic perspective (Bakhtin, 1977;

1929/1998; 1986/2003), helps me to show descriptive elements of the phenomenon. Since data analysis is ongoing, preliminary results will be presented.
Second, situating ourselves in the interpretative paradigm of qualitative research, I will emphasize the few advances in methodological terms that are carry out by this study. On this point, attention will be given to the position of the researcher in this context. From some readings, including Martin Heidegger's *Age of the World Picture* (1949/1986), and from the methodological design presented, a discussion could be initiated on the direction and scope of qualitative research in human sciences, and its potential to the research community concerning history of mathematic and the teaching-learning of mathematics.

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René Guitart (France): ***History in Mathematics According to André Weil***.

André Weil is one of the main mathematicians in the XXth century, and he worked also on the history of number theory.
Furthermore he used history to explain the meaning of his mathematical work to his sister, the philosopher Simone Weil. He did that in two letters of 1940.
In the first letter he wrote that the mathematician is an artist, similar to a sculptor, working in a very hard matter, namely the mathematical culture, where the constraints are the previous theories and problems. He suggested to examine history of mathematics from this point of view.
In the second letter to Simone, he explained the meaning of his own work, with a lot of technical details. Mainly he claimed to develop and to construct a triple analogy, between three mathematical domains in progress : the theory of numbers and fields of numbers, the riemanian theory of algebraic functions on the field of complex numbers, the theory of functions on Galois'fields. These theories are described from a historical point of view. He considered that what he constructs is a kind of trilingual dictionnary, in order to decipher a trilingual text made of desultory fragments.
We will show that this way to give sense to mathematical activity in narrow relation with history is useful for mathematicians, teachers and students.

Leo Rogers (UK): ***Historical Epistemology: Contexts for contemplating classroom activites***.

In an earlier paper (Rogers 1995) I discussed the teaching of mathematics in a critique of the formalist paradigm that persisted in classrooms and can still be found in textbooks and teachers' manuals today. Notwithstanding the development of new historiography we have little change in the presentation of the History of Mathematics in texts for teachers, and even in prestigious handbooks such as (Harel & Sowdwer 2007: 811-820) where a totally internalist and anachronistic view of history purports to show the historical-epistemological basis for proof in mathematics. Mathematical Knowledge cannot simply be reconstructed 'as such' and recent publications now begin to show the detail and contexts of early mathematics e.g. (Katz 2007); that heretofore was the domain of mere speculation and mis-interpretation in some didactic materials. There must also be a parallel programme of reconstructing what the agents thought were permissible or recommendable steps, and how they understood such concepts as knowledge, evidence, observation, representation, objectivity, and proof.
Historical epistemology is an important tool to address these second-order considerations. However, any analysis of the development of mathematical ideas necessarily calls on a serious approach to the social and cultural contexts in which the mathematical knowledge was generated and should attempt to answer questions about the motivations and means (practical and theoretical) available to the agents involved.
By taking Examples from Galileo's arguments on the Third Day of the *Discorsi* on the progression from 'uniform' to 'naturally accelerated' motion Grosholz (2007) shows how the heuristic device of Productive Ambiguity and the contexts of historical epistemology are relevant to contemporary

pedagogy. Further examples from Chemla (2012), Keller (2006), Netz (1999) and Robson & Stedall (2009) on the writing of history and the growth of mathematical knowledge at different times located in their contemporary contexts can be used as case studies for an epistemological approach to classroom activities.

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Room A212:

Aline Bernardes &Tatiana Roque (Brazil): ***Reflecting on meta-discursive rules through episodes from History of Matrices.***

In our country, the most of Linear Algebra courses starts with the notion of matrix as a mathematical object independent of other concepts. However, historically, the notion of matrix emerges and develops associated with other concepts such as determinants, linear transformations and bilinear forms. Actually, among the above concepts, the notion of matrix was the last to emerge.

In this work, we intend to present our experiment in conducting workshops with graduate students in mathematics using episodes in the history of matrices, based on Brechenmacher's historical interpretation (2006). Our objectives with these workshops are: i) to develop students' historical awareness on the different meanings attributed to the concept of matrix and the formation of linear algebra as a discipline, and ii) to stimulate students' reflection on meta-discursive rules related to matrices and linear algebra.

The conceptual framework we have been used is based on Sfard's theory of thinking as communicating (Sfard, 2008) and on the concepts of epistemic objects and epistemic techniques (Epple, 2004). We have been inspired by the works of Kjeldsen (2011) and Kjdeldsen & Blomhøj (2012) for this conceptual framework.

According to Sfard, mathematics is seen as a type of discourse governed by certain rules and learning mathematics means to engage in this discourse. Two important types of rules for the learning of mathematics are object level rules and meta-discursive rules or meta-level rules: the first relates to the properties of mathematical objects, the latter, of most interest to this work, are those that govern and shape the mathematical discourse and are usually implicit in the discourse. The concepts of epistemic objects and epistemic techniques will be useful to distinguish the objects under investigation in an episode of research and the tools used to answer questions about these objects (Epple, 2004).

We are using the above concepts to design workshops on the history of the matrices for undergraduate students in mathematics.

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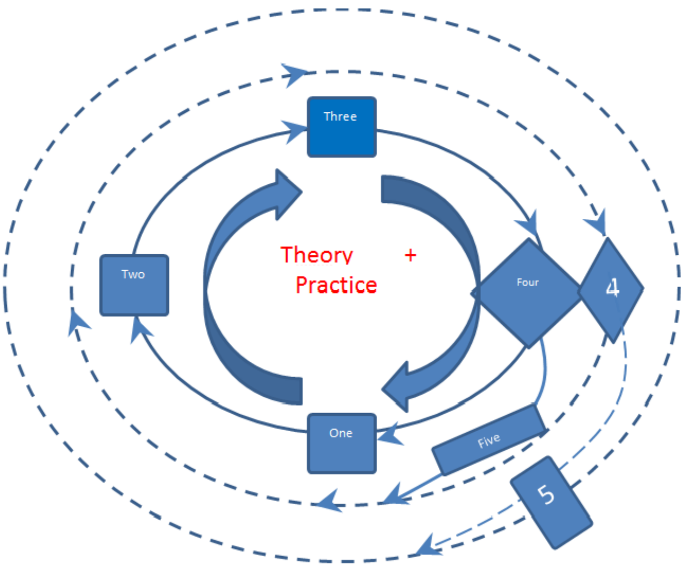
Ke Wang (China): ***A New Paradigm in the Field of HPM - Design Research.***

Based on the situation that research field of HPM is facing the lack of theoretical basis and research methods, this paper primarily proposes design research in the field of HPM, defines its conception in HPM, describes the five steps and main features, models the design research. (fig.1) What's more, it discusses the great practical and theoretical significance of design research in the field of HPM, and first proposed the hermeneutical circle of educational research,(fig.5) creatively establish a theory on the development of HPM: triangular pyramid model,(Fig.3) and analyze the future challenges in the HPM.



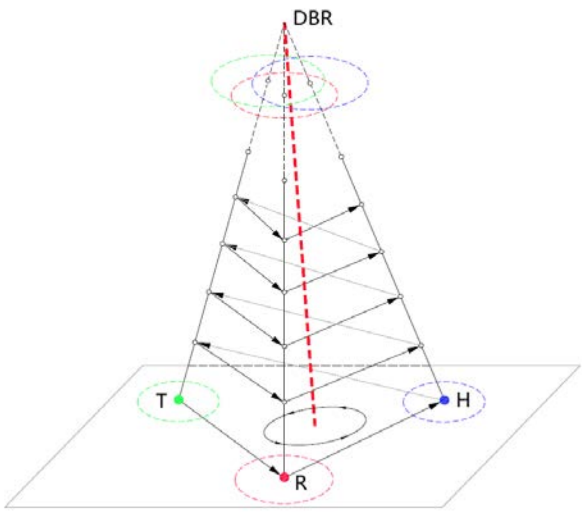


Fig.1 The five steps of design research on HPM

Fig.2 The triangular pyramid model

1. Research & Preparation (History on knowledge point, Teaching Problem, and so on)
2. Development & Design (How to design the Instruction based on HPM)
3. Implementation & Operation (Implement the teaching on HPM)
4. Analysis & Assessment (Analyze the data which collected in the teaching)
5. Popularization & Application (Popular the design on HPM)

As shown in the Fig. 1, there are more than three cycles from step one to step four in the first solid circle, and adjust the design on HPM to popular it in others' classroom.

The solid circle: the circle of design research.

The green broken circle: the field of knowledge on Teacher.

The red broken circle: the field of knowledge on Researcher.

The blue broken circle: the field of knowledge on historian of mathematics.

DBR(red dotted line): design research on HPM.(Fig.2)

Firstly, the problems are realized by teachers (T) in their teaching, and then, they are fed back to Researchers(R). Finally, researchers integrate the history into teaching, the cycle will carry on. There are several circles in design research in order to solve the problems, eventually a good teaching design on HPM comes out. With more and more cycles in the research process, each of the three fields will be improved to be a higher level, and the three fields will be integrated, lastly into a whole. But the vertex and bottom of pyramid are desired state, may not be existed in real research.

The three field of knowledge on T, R, and H will be illustrated in the following.

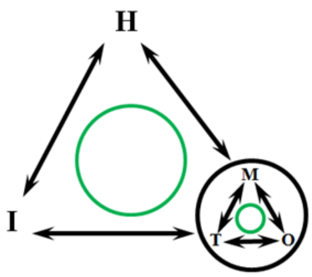


Fig.3

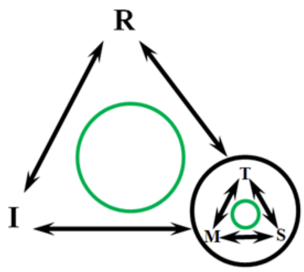


Fig.4

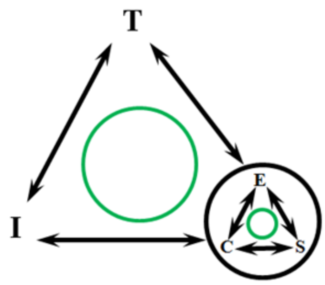


Fig.5

Hermeneutical circle on history of mathematics: Old mathematician (M) set up mathematical theories (T) by interpreting mathematical objects (O), M, O and T form the first circle. Historians of mathematics (H) interpret the first circle by supposing to go into the old mathematicians' world, lastly, get a interpreted result---history of mathematics (I).(Jahnke H N.1994)

Hermeneutical circle on mathematics teaching: Editors (E) compile textbook (C) by interpreting the curriculum standard and mathematical content knowledge (S), E, C and S form the first circle. Mathematics teachers (T) interpret the first circle by supposing to go into the world of editors, and get a pedagogical content knowledge (I). (Yiwan Su, 2004, Wann-Sheng Horng, 2005)

Hermeneutical circle on educational research: Teachers (T) teach mathematics (M) by interpreting student(S), T, S and M form the first circle. Education researchers(R) interpret the first circle by going into a real teaching situation to get a method of solving the teaching problem (I).

Karel Zavřel (Czech Republic): ***Parallels between phylogeny and ontogeny of logic.***

The principle of a parallel between the ontogenetic and phylogenetic development is an accepted principle in mathematics education. If teaching omits some of the stages, that were important in the historical development of the topic, it can become an obstacle for students understanding. Logic is in this respect a very special discipline. Its origins are linked to ancient philosophy; during the Middle Ages it became an instrument for theological disputations, thus most of its history can be placed in the area of the humanities. It was not until the late 19th century that a specific kind of logic, which we today call mathematical, started to develop. Later logic entered also natural sciences and engineering, where it is an indispensable tool in areas such as artificial intelligence.

Despite of this, the teaching of logic, at least in the Czech school curriculum, starts with this younger logic, the Boolean truth-functions and Fregean quantifiers (in the first year of upper secondary education). The previous two thousand years of development of this fascinating discipline do not enter the classroom. The teaching of logical connectives and quantifiers is often very formal. It is not based on the student's previous experiences, but it is presented only as knowledge and skills that must be learned.

The ordinary language that student use is ambiguous and context-dependent; accent and intonation have an important role for the understanding of sentences. Its implicit logic is usually modal. In logic classes, however, suddenly everything that we mentioned becomes irrelevant. Important is only the logical form that can be filled almost with any content.

In our work we try to find out whether there are any parallels between the historical development of logic and the spontaneous growth of logical thinking in children. We are trying to discover errors, which appeared in the phylogeny of logic and which can be found also in the logical reasoning of children. For comparison, in this respect we also use results of studies of the reasoning of aborigine peoples.

We study the logical thinking of children in areas where there is a parallel to the historical material. For example questions like the syllogism, classification, negation or implication. We concentrate especially on students leaving lower secondary education and we are comparing their level of logical thinking with the requirements that will be imposed on them in the first year of high school. At the same time we try to map the development of logical abilities during the entire lower secondary education. In this period, according to Piaget, there should be a transition between two stages of cognitive development: namely the stages of concrete and formal operations. The stage of formal operations, which according to Piaget begins around the 12th year of age, should be especially noticeable by the ability of children's logical thinking.

Room A214:

Katrin Reimann (Germany): ***On the understanding of the concept of number in Euler's algebra.***

Understanding the concept of number and variable is very important for the understanding of mathematics in general and algebra in particular. Many problems encountered by students during the time of transition from arithmetic to algebra are based on different usages and understandings of numbers and variables.
A look at the historical development of algebra is helpful to gain insight into the understanding of mathematics and the nature of the discussed objects. This insight could be used to understand the occurring problems for the students when learning algebra.
In my presentation I will discuss the concept of number by Euler in his textbook "Elements of Algebra" from 1770. Unlike modern mathematics, where numbers and variables are seen as abstract objects, Euler considers numbers as objects grounded in an empirical universe of discourse. Numbers are defined as the ratio of measurable quantities, such as money, length or weight. The motivation and explanation of new characteristics of numbers and also for the initiation of a new number range are based on the characteristics of the underlying empirical quantities. An abstract formal introduction does not occur. Thus Euler's approach in this textbook can be described as inductive. The complex numbers represent an exception as they appear as a result of the operation "extraction of a square root", but do not have a reference object. Euler is in the position to operate with complex numbers by transferring the well-known operations to the new symbols. Nevertheless, he sees complex numbers as impossible numbers. Therefore, the occurrence of complex numbers indicates for Euler the impossibility to find a solution for the problem. Using the well-known operations on complex numbers without any further discussion is from my point of view problematic, which will also be exemplified in this oral presentation.
Euler's understanding of variables is based on the concept of number. He handles variables as if they were concrete numbers. The basic arithmetic operations - addition, subtraction, division and multiplication - which were initially introduced for numbers, are transferred to variables.
The second part of Euler's textbook discusses equations and linear systems. The rules to manipulate equations are not formally defined, just as well as the operations for higher linear systems are only mentioned without any abstract introduction.
In conclusion, the presentation aims at analysing and discussing Euler's understanding of algebra with the help of the philosophical and didactical tools provided by the idea of empirical theories (cp. Stegmüller, Hauptströmungen der Gegenwartsphilosophie, Bd.2 1987, Burscheid/Struve, Mathematikdidaktik in Rekonstruktion 2010).

Gabriela Buendía Abalos (Mexico): ***The necessity of developing epistemologies of practices and uses so they can conform a significances basis for any kind of didactical proposal.***

Within the paradigm of social construction in mathematics education, the theoretical framework of Socioepistemology seeks for an epistemological explanation in which social practices are considered as generators of mathematical knowledge. This social nature epistemology provides historicity to the teaching mathematical concepts usually presented as universal objects in time and space; it enables observation of differences between the scientific knowledge and the taught one; and allows us to understand what norms the evolution of scientific knowledge. So, we have to develop research strategies to give evidence, through an historical analysis, of the circumstances surrounding knowledge construction. The kind of questions that guide this socio-epistemological analysis are like, why was done what has been done while dealing with certain mathematical knowledge? Or, why do know or teach the way we actually do it? In any case, the aim is to question the actual school mathematical knowledge considering its social origin context, in particular through history. So it is a search for the tools, artifacts, and arguments involved in its generation always taking into account that they are socio-culturally situated in a historical context and considering not only the mathematical result -as in a historical inform of the mathematical object evolution- but the human activity developed and that surrounded it. Under this theoretical framework, the kind of knowledge that matters is not the product of a logical structure or the knowledge seen as a set of objects that acquire a pre-existence nature in school mathematics. Considering the social nature of mathematics construction and the epistemological role that social practices have in that construction, the knowledge that matters is the knowledge-in-use. We are dealing with the necessity of developing epistemologies of practices and uses so they can conform a significances basis for any kind of didactical proposal: class activities, text books, curricular reforms, and so on.

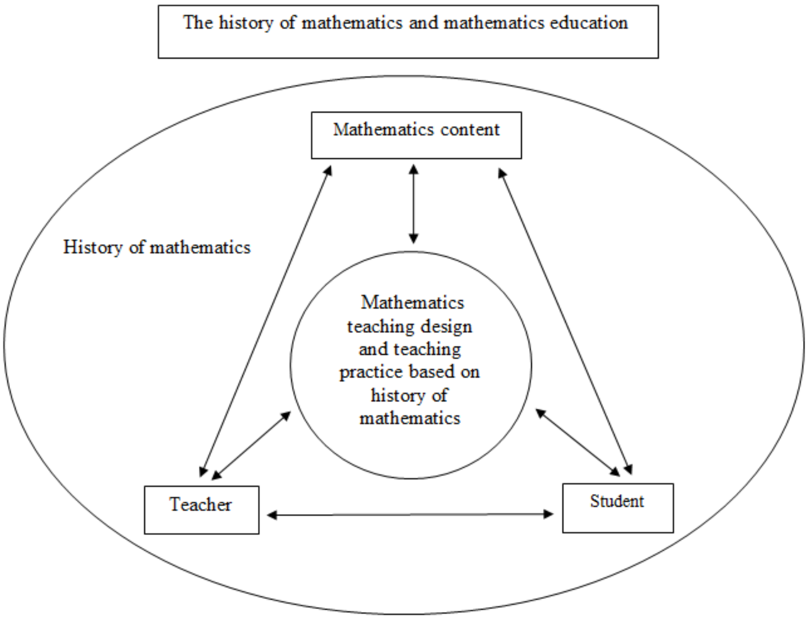
In particular, this short communication intends to present elements of a socio-epistemology for graphics. This epistemology of practices and uses propose graphing as a reference practice that develops through a reference context -as school or a historical context like the physics mathematization- favoring a systemic development of mathematical tools. Understood as a practice, it favors the execution of multiple tasks and the adjustments of its own structure to generate patterns and generalizations. Additionally, it acts as a means for the development of reasoning and arguments. All of this is involved in the role graphing plays in regulating the meaning and functionality of mathematics. Thus, it can be recognized that graphs have had trough several historical episodes different uses and these uses evolve in a dialectic relation with different situations faced in the development of scientific thinking. It is important to understand these uses and to know how to analyze them in order to explain their role in the construction of a significant and articulated mathematical knowledge.

Lin Jiale (China): ***A Theoretical Framework of HPM.***

Today, the relationship between the history of mathematics and mathematics education (HPM) is a special research field in mathematics education. There are three core issues in this field: 1. Why is HPM studied? 2. What are studied on HPM? 3. How can HPM be realized? For the first question, some scholars have made a reply from the perspective of the teachers and students respectively; for the third question, some scholars have established the theoretical framework of how to integrate the history of mathematics into mathematics teaching. However, the second question is still to be discussed.

In this paper, we try to establish a theoretical framework of the relationship between history of mathematics and mathematics education on the basis of the triangle model of teaching. What's more, we discuss the first and second questions based on this framework and try to give comprehensive and holistic answers as far as possible.

Key words: history of mathematics; mathematics teaching; theoretical framework



Room A104:
Ingo Witzke (Germany): ***On preservice teachers' understanding of mathematics.***

Around 250 preservice school teachers from German universities (Cologne and Siegen) have been asked in a survey for their retrospective view on their way from school to university mathematics. Surprisingly, the systematic qualitative content analysis of the data shows that to a substantial extent, students have more problems with a feeling of "differentness" of school and university mathematics than with the abrupt rise in content-specific requirements. This coincides with the author's results of a reconstructive comparative analysis of school and university textbooks. Preservice teachers seem to distinguish clearly between school and university mathematics with respect to its *nature.* Following the ideas of constructivism, this has a lot to do with fundamental differences regarding the *nature of objects* we deal with in classrooms and in lecture halls: As much of university mathematics for good reasons is about abstract entities, most of school mathematics for good reasons is bounded to concrete objects. A. Schoenfeld proposes in this context that pupils acquire at school an *empiricist belief system*, whether students at universities are confronted with a rather *formal(istic) view* on mathematics. In the terminology of A. Sierpinska, students at this point have to overcome a variety of "epistemological obstacles", requiring a big change in their understanding of what mathematics is about.
These findings are essential for the author's design of a course for preservice teachers, which will be implemented in Summer 2014 for the first time. The course aims at making students aware of crucial changes regarding the *nature of mathematics* on their way from school to university (by discussing transcripts, textbooks, standards, historical sources etc.). Central is the historical and philosophical development of mathematics especially at the beginning of the last century. The plan is to take advantage of the idea that students on an individual level have to overcome similar problems as mankind had in history: the author is convinced that on an epistemological level, students can learn e. g. from aspects of the foundational crisis of mathematics at the beginning of the 20th century.
The discussion of selected examples of historical examples in the course taken from Euclid's Geometry, Leibniz' Differential Calculus or Hilbert's Foundations of Geometry thereby serves two purposes. *On the one hand* it helps to understand why nature of mathematics changed dramatically in history from *empiricism* towards *formalism* – giving undergraduate students crucial

insights regarding their individual development between these poles. *On the other hand* history provides us with a variety of beautiful pieces of substantial mathematics which was developed in an *empirical* manner (describing the physical world around us) providing prospective teachers with material which shows how to do it right in school.
Taking all this into account it is the author's aim to raise awareness for epistemological obstacles caused by different understandings of mathematics. This is connected to the hope that the knowledge about these obstacles will help more students to develop an adequate perspective regarding the *nature of mathematics* in classroom *and* lecture hall.

Arto Mutanen (Findland): ***Knowledge acquisition and mathematical reasoning.***

Mathematical, and logical, reasoning can be understood as tautologous (see *Tractatus* 6.1231). This tautologousness makes the reasoning informationally empty. Mathematical and logical truths are valid, i.e., true in every possible world. That is, mathematical and logical truths do not exclude any possibilities, and contradictory statements exclude all of them. This is another way to characterize the informational emptiness of mathematics and logic. The informational emptiness is not the whole story. Mathematical knowledge is very important in itself and because of its applications. Still, the problem is how mathematics (and logic) can increase our knowledge. To understand how mathematics increase or knowledge it is important to analyze concrete mathematical reasoning. In geometry the essential thing is the constructivity of the whole reasoning process. (Hintikka and Remes 1974) More generally mathematical reasoning can be characterized as constructive. In fact, Russell and Carnap can be seen as examples of this constructive approach. The constructivity is a key notion in understanding the mathematical knowledge acquisition. The constructivity is closely connected to the methodology and epistemology of mathematics. The constructivity can be identified at two different but related levels: at the level of inference steps and at the level of inference strategy. However, at the same, the constructivity allows us to understand the applicability of mathematical reasoning to experimental and empirical reasoning. The strategies of experimental and mathematical reasoning are parallel. (Hintikka 2007) In the present paper we will consider the character of mathematical knowledge; what kind of knowledge it is and how mathematical reasoning increases our knowledge. Eventually we will consider how this could – or should – be taken into account in teaching of mathematics and natural sciences.

Wang Xiaoqin (China): ***HPM in Mainland China: an Overview.***

Five themes on HPM are focused in Mainland China:

● Discussion on "Whys" & "Hows".

The "whys" are categorized corresponding to three mathematics teaching objectives, and four approaches to using the history of mathematics in teaching are identified based upon teaching practice.

● Education-oriented researches on the history of mathematics.

The history of specific topics on school mathematic are studied, for example, the history of the concept of ellipse, the history of using the letters to represent the numbers, etc.

● Empirical studies on the "historical parallelism".

For example, students' understanding of the concept of the tangent line is surveyed, and the historical parallelism id examined.

● Integrating the history of mathematics into mathematics teaching: classroom practice or experiments.

Many teaching experiments are carried out and many teaching materials have been built so far.

● HPM & mathematics teachers' professional development.

In this lecture, we mainly focus on the four approaches to using history in teaching based on some lessons, such as the linear equation with one unknown, the application of similar triangles & congruent triangles, the concept of complex numbers, etc.

Room A130:

Klaus Volkert (Germany): *The problem of the parallels at the 18th century: Kästner, Klügel and other people.*

During the 18th century a lot of work on the problem of the parallels – that was in the traditional understanding, proving Euclid's parallel postulate on the base of his other axioms and postulates – was done. There was G. Saccheri with his remarkable work "Euclidis ab omni naevo vindicatus" (1737) – which remained more or less completely unnoticed - on one hand and A. M. Legendre with his widespread demonstrations ("Eléments de géométrie" (1794)) on the other hand. But in between there was also a remarkable dissertation written by Georg Simon Klügel ("Conatuum praecipuorum theoriam parallelarum demonstrandi recensio" (1763)) under the guidance of Abraham Gotthelf Kästner at Göttingen; it was the latter who draw some skeptical conclusions of the work done by Klügel in his remarks in his "Anfangsgründe" (first published 1758 – 1764; there are different later editions).

In my conference I want to retrace in a short way the history of the problem of the parallels in particular the proposals made by Saccheri and Legendre. It main purpose is to describe Klügel's critical work and the conclusions derived from it by Kästner. We will see that the convictions of the mathematicians were surprisingly enough very important in this history – the main difference between J. Bolyai and N. Lobachevsky on one hand and there precursors laying exactly in this respect, Klügel and Kästner being remarkable forerunners of them.

The dissertation by Klügel is now available in a German translation by Dr. M. Hellmann (Weilheim) in Volkert, K. "Das Unmögliche denken. Die Rezeption der nichteuklidischen Geometrie im deutschsprachigen Raum 1860 – 1900" (Heidelberg: Springer, 2013).

Thomas Hausberger (France): *Training students in mathematics education to use history and epistemological tools: theory and pratice on the basis of experiments conducted at Montpellier University (France).*

The 2010 reform of initial teacher education in France has been an opportunity to introduce history and epistemology courses in the master's degree program in mathematics education. I will give an account and draw conclusions based on my experience as University teacher in history and epistemology of mathematics as well as researcher in mathematical education at Montpellier University (France). Both theoretical perspectives and practical aspects will be presented.

At Montpellier University, 2 courses of 50 hours each are devoted to history and epistemology of mathematics. The first course focuses on tertiary level mathematics and aims at promoting reflexive thinking on mathematical objects and methods in order to make explicit the underlying epistemologies and the cultural background for a deeper understanding of mathematics. Students are trained to analyze and comment a corpus of documents including primary sources or essays written by historians or philosophers. The second course is directed towards secondary level mathematics with a view to articulating history and epistemology of mathematics with didactics of mathematics. Practical aspects include learning to use history and epistemological tools in the classroom. It leads to the conception of pedagogical scenarios and their implementation, since second year Master students already have a teaching duty of 6 hours a week. The central role of epistemological analysis in didactics is acknowledged by the French school of didactics of mathematics since its foundation. Combining a historical and epistemological analysis of the genesis of mathematical concepts with a didactical analysis of the teaching and of the difficulties of the students has proved to be an efficient method to gain a better understanding of these difficulties and to build innovative activities. I will elaborate on the interplay between epistemology and didactics, building on the literature in didactics of mathematics and my experience as University teacher and researcher in mathematical education, but restricting on the aspects that may be considered helpful to support teacher competence development. Finally, I will give examples of student's productions of pedagogical scenarios that either use historical sources and historical perspectives or use epistemology as a tool, for instance through the meta lever, that is the use in the classroom of knowledge *about* mathematics (or, compared to mathematical knowledge itself, information about what constitutes mathematical knowledge, in other words, an epistemological clarification). I will also report on the difficulties that the students encountered in the engineering of these activities and their experimentation, and also on the position of the instructor that supervised the work.

Cecilia Costa José Miguel Alves & Marta Guerra (Portugal): ***The ancestral Chinese method for solving linear systems of equations seen by a ten years old Portuguese boy.***

Inspired by the simplicity of the method for solving linear systems of equations existent on the Chinese ancient mathematics book "The Nine Chapters on the Mathematical Art", we decided to test the viability of a young child understand the method, reproduce and apply it to solve problems that involve the resolution of a system of linear equations.
As the referred Chinese method, only involves elementary arithmetic calculations we decided to do a case study with a ten years old (Portuguese) boy. He has just concluded the elementary school, had good marks and talent to math. Notice that at this time, the boy doesn't know how to solve even equations of first grade.
We created a task (partitioned in three parts), transposing didactically, the Chinese ancient method for solving linear systems of equations to be presented to a young boy. The three parts have different learning objectives. First, we present the Chinese ancient method to the child, and then he solved a problem with the help of one of the team researcher. Secondly, the boy solves a similar problem, previously recalling the method he had learned a few days before. Some days after, in a different context, intentionally created to test if the child would be able to apply by himself the Chinese method, he solved a problem/challenge that needed the resolution of a linear system of equations.

By research design, the boy solved the task each part in a different time; outside the classroom and without relation with his classes; in a familiar context as a mathematical challenge as he does some times.
We videotape the three sessions of the presentation and resolution of the task and an interview to the young boy after solving the last part of the task. We keep the productions of the young boy.
The analysis of the collected data allows us to affirm that this young boy was able to use and apply this method in non-didactical situations, solving problems that involve the resolution of systems with three equations and three unknowns and also with four equations and four unknowns.
In Portugal the Gaussian elimination to solve linear systems of equations – similar to the Chinese ancient method – is taught in first year of university.
In our opinion, the Gaussian elimination requires the use of algebraic language in the formal writing of systems of equations, while the Chinese method as far as we know only uses double-entry tables (analogous to current matrices) and arithmetic operations. This would mean that it is possible to teach much earlier the Chinese method, than it happens with the Gaussian elimination.
In this oral presentation we pretend detail the case study done, presenting the task, the productions and some answers of the young boy. We would like also to reflect on the study done.

Thursday July 17

<u>18:00-19:00: Oral presentation: **Theme 4, 5, 6 & 7**</u>

Room A303 (themes 5):
James F. Kiernan (USA): ***A Course to Address the Issue of Diversity.***

Brooklyn College is now offering an upper level CORE course entitled "The Mathematics of Non-Western Civilizations". The course is offered to all students who have completed a number of lower level CORE courses. The original course designed by Jeff Suzuki was based on readings from Katz' *Sourcebook* (2007). Several instructors in the department have now taught the course. Since the recent publication of a third edition Joseph's *The Crest of the Peacock*(2011), I have successfully used it as the main text in the course .Though the use of this text students become acquainted with concepts such as Eurocentism and Ethnomathematics in addition to learning some elementary mathematics developed in civilizations which have been frequently neglected. This talk will discuss issues regarding teaching a course on the diversity of mathematical development to a diverse population with diverse abilities. Examples of syllabi, assignments and assessments will be provided. I hope that this presentation will lead to some fruitful discussion of using history of mathematics in your classroom.

Andrea V. Rohrer (Brazil): ***The Teuto-Brazilians of Friburgo – mathematics textbooks and the use of (non-metric) measure systems.***

During the 19th century, and until the 1930s, the German immigration in Brazil had brought about eight thousand Germans to live in the state of Sao Paulo. (Kahle, 1937, p. 29; Miranda, 2005; von Simson, 1999, 1997, pp. 63-65)
According to a report from 1852, written by the district administrator of the rural village of Bernkastel (140 km south of Bonn, in Prussia), the production of goods had not increased at the

same speed as the growth rate of population. Due to the predictable nourishment difficulties, already by 1846, 633 persons had emigrated either to USA or Brazil (MoZ, 2007).
Meanwhile, a series of intense debates and liberal movements sought to democratize and build a new republic in Brazil, purging all remaining connections to the Portuguese monarchy. Many members of these movements, owners of coffee plantations, had rejected slavery and had started to properly hire laborers, paying them salaries. One of the crucial figures of this liberal movement was senator Nicolau de Campos Vergueiro who, in a partnership with the owner of the *Fazenda Sete Quedas*, brought, in 1856, 112 German families to work in the fields (Di Francesco, 2007, p. 25; dos Santos Bezerra, 2002, pp. 69ff.; Leite, 2006; Fortes, 2003).
Once the immigrants had evened up their debts with the landowners, they were able to save money and move away; some of them decided to buy land lots in the nearby region and cultivated coffee, potato, beans, etc. (von Simson, 1997) Eight families, among these 112, had settled, between the years 1864 and 1877, in a rural region situated in the southwest of Campinas and north of Indaiatuba. They were immigrants from the German regions of *Rheinland-Pfalz* and *Schleswig-Holstein*, and from the canton of Bern in the German Switzerland. In 1879 the district of *Friedburg*, later called *Friburgo*, was founded and a school was opened this same year in October (Guebel, 1937, p. 2).
In general, the first German colonies were founded in very remote geographic locations, which had caused a need for opening schools inside the same colony. These schools had been established with German curricula, and were, for a long period of time, the mainstay for the transmission of the German traditions and identity, difficulting the integration process (Schubring, 2003, p. 14).
During this presentation, we would like to present the results of an ethnographic research undertaken in *Friburgo*, in 2007. In particular, we would like to show that, despite the promulgated laws obligating the teaching in the national language (1920), *i.e.* Portuguese, and the use of the International System of Units (1872), *i.e.* the metric system, the school of *Friburgo* had continued to use mathematics textbooks, edited in German language, which also included conversion tables of non-metric measure systems, until after the 1st World War. As we will see, some non-metric units remain in use, at least orally.

Room A212 (theme 6):
Jérôme Auvinet (France): ***Around a book dedicated to childhood friends: L'Initiation mathématique of C.-A. Laisant.***

In 1906, the French mathematician Charles-Ange Laisant (1841-1920) publishes *L'Initiation mathématique*. This work offers through 65 lessons an original introduction to mathematics addressed to young children and above all their educators, from the bases of counting to an approach of more complex concepts (functions, conicals, sums of integers for example). It is based on various manipulations of mathematical objects and on experimentation in general, and it appears quite successful: it is reissued many times (and also translated into Italian). This small book is the first of a collection directed by Laisant, called "*Les Initiations scientifiques*" that deals with astronomy, chemistry, etc. ... It enables its author, former polytechnic student who first interests in preparatory education, to start a new itinerary linked with his political and social thinking. It

illustrates his progressive involvement to a movement known as "L'Éducation nouvelle" and his relations with pedagogues such as the Spanish Francisco Ferrer.
We first present some specific mechanisms used in these lessons in order to study the practice of mathematics exposed here. We highlight the didactical or mathematical influences that appear in this project, from Pestalozzi to Jean Macé or Édouard Lucas. We also look into the genesis of that work in the context of the time, its reception and the principles that are applied and finally the debate in which Laisant participates thanks to "*L'Initiation mathématique*".

Thomas Morel (Germany): ***Teaching mathematics in mining academies: an overview at the end of the 18th century.***

At the end of the 18th century, many mining academies were created, mostly in the German speaking countries, to improve the overall scientific education of mining technicians and officials. While historians of science have so far dedicated studies to chemistry and mineralogy, mathematics has been vastly overlooked. It is usually assumed that mathematical sciences were taught and used uniformly and at an elementary level in the different *Bergakademien* of Freiberg, Schemnitz, Clausthal, etc.
In this talk, I intend to show that there was in fact a great variety of approaches in mathematics teaching in these institutions, ranging from academic *Gelehrsamkeit* to the practical *Brauchbarkeit* of mining engineers. Some of the sources used are the archive of the various *Bergakademien* as well as the textbooks published by mathematics professors, but also manuscripts, handwritten lectures and test reports.
The example of the Freiberg mining academy, where mathematics teaching reached a very high level at the end of the 18th century, proves that these disciplines there were not intrinsically inferior, but only different from the university lectures of that time. The subterranean geometry (*Markscheidekunst*) will then be used as an example to illustrate the originality of this scientific and technical teaching tradition.

Room A214 (theme 6):
Luis Puig (Spain): ***The beginning of algebra in Spanish in the sixteenth Century: Marco Aurel's Arithmetica Algebratica.***

The book *Arithmetica Algebratica*, published in Valencia in 1552, is the first printed book on algebra written in Spanish. However its author, Marco Aurel, had not Spanish as his mother tongue, he was a German residing in Spain.
A German working in Spain and writing in Spanish in the sixteenth Century is not a so strange fact. When Marco Aurel's book is published, the King of Spain, Carlos I, being also heir of the House of Hausburg and the House of Valois-Burgundy, has become, as Karl V, the Emperor of the Holy Roman Empire, and the Archduke of Austria, and is ruling then over German territories. This is the cultural and political context in which the book is published, and these are the features that make singular Marco Aurel's book. But the reason why we are especially interested in this book is the fact that it is the only book using the German cossic signs written in Spanish, and published in Spain. Algebras written or published in Spain during the sixteenth Century were rethorical or, when syncopated, they used Italian abbreviations.

Marco Aurel's book gives us the only opportunity to study in a Spanish text the way in which the use of cossic signs shapes algebraic reasoning's.
To carry out this study we have examined the intertextual relation of Marco Aurel's book with other books on Algebra, that dialogue with it, by referring explicitly or implicitly to it, by being its contemporaries in Spain, or by being the result of readings of Marco Aurel's book.

Dirk De Bock and Geert Vanpaemel (Belgium): ***The Belgian journal Mathematica & Paedagogia (1953-1974): A forum for the national and international scene in mathematics education.***

In 1953 the Belgian Society of Mathematics Teachers was founded. The Society brought together a few hundred mathematics teachers from both linguistic communities (French and Dutch). It started its own professional journal *Mathematica & Paedagogia* (M&P). Willy Servais, the Society's first president (until 1969), served on the editorial board of M&P and became the journal's figurehead. Servais was an open-minded, inspiring personality and an unconditional proponent of international exchange in mathematics education (Vanpaemel, De Bock, & Verschaffel, 2012). Already in his first M&P *Editorial* he strongly pleaded for "opening the journal's columns to our colleagues of other countries" (Servais, 1953). Many leading mathematicians and mathematics teachers, but also psychologists and pedagogues contributed to the journal. In the 1950's M&P became an international and interdisciplinary forum of ideas, in particular for members of CIEAEM, the *International Commission for the Study and Improvement of Mathematics Teaching*, of which Servais was one of the founding members (Bernet & Jaquet, 1998). Although many of the papers in M&P were concerned with the ongoing New Math reform, the journal also published special sections on the applications of mathematics, the history of mathematics and "culture mathématique". The journal ceased publication in 1974 as the Society split in two linguistic sections (Miewis, 2003).
In this presentation we will present a close reading of some key contributions in M&P by the leading international actors of that time (C. Gattegno, F. Lenger, E. Castelnuovo, J. L. Nicolet, T. J. Fletcher, G. Choquet, Z. Krygowska, M. Puig Adam, G. Choquet, H. Freudenthal, …), and analyze the reception of their ideas in the local Belgian community of mathematics teachers.

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Room A100a (theme 6):

Karel Lepka (Czech Republic): ***T. G. Masaryk and mathematics.***

Tomáš Garrique Masaryk (1850-1937) was the first President of the new established state –

Czechoslovakia. He was not only the statesman, but he was also professor of sociology at Charles University in Prague. He strongly supported the founding of a university in Brno, and therefore the university in Brno now bears his name.
Masaryk was not involved in mathematics as a scientific discipline, but there is a connection between him and mathematics. Masaryk founded the journal *Athenaeum*, which was published from 1883 to 1893. *Athenaeum* was a journal for literary and scientific criticism and we can see mathematical problems on its pages.
There were published reviews about mathematical publications, above all mathematical books. Leading Czech mathematicians were among the reviewers and some reviews were written by Masaryk. Except reviews we can find even articles dealing with mathematics, for example the article "Základové ryze arithmetické theorie veličin" ("Principles of the purely arithmetic theory of quantities") published by Matyáš Lerch in Volume 3 of *Athenaeum*, pp. 223-236. In this article, he derived integers, rational and real numbers on basis natural numbers in a purely arithmetical way.
In 1886 big controversy as to whether the allegedly very old manuscripts are genuine arose in Czech scientific society. In Athenaeum, vol. 3, 299-307, August Seydler (1849-1891) published an article, in which he tried prove that manuscripts are fake. He used probability and calculated that probability of the fact, that manuscripts are true is extremely low. This article is not only one on the first papers from theory of probability, but even one of the first mathematical papers which has practical use in the humanities.

Mária Cristina de Almeida (Portugal): ***Mathematics textbooks, in Portugal: the case of the first unique mandatory Algebra textbook (1950).***

Textbooks are some of the most relevant elements of the material culture of any discipline. Textbooks are sensitive to national contexts and can be seen as probes of the state and structure of mathematical education, its goals and its organization.
In Portugal, by 1948, an educational reform established that a disciplines' textbook for each grade of secondary schooling would be the same for all of the schools (*Liceus*). This new system relied on a tender for the approval of the unique mandatory textbook that would be used by teachers and students the following five years. The tender was issued by the Ministry of Education and encompassed a review of all the textbooks that were submitted to it. Regarding to mathematics textbooks, there was an approved textbook for each topic studied within a school year – Arithmetic, Algebra, Geometry, Trigonometry. The emergence of a unique mandatory textbook was not peaceful; some teachers published their arguments against this system in teachers' bulletins, as well as, in the press. The first mathematics textbook to be approved was an Algebra textbook, for the first grade of upper secondary school, and it was object of dispute in the mathematics journal *Gazeta da Matemática*. This paper will study this dispute and presents an analysis of content of two edition of the first unique mandatory Algebra textbook approved in 1950.

Room A104 (theme 7):
Kajsa Bråting (Sweeden): ***E.G Björling's view of fundamental concepts in mathematical analysis in an epistemological and didactical perspective.***

In this talk I consider the Swedish 19th century mathematician E.G Björling's (1808-1872) view off fundamental concepts in mathematical analysis. The aim is to discuss problems that may arise when mathematical definitions are too vague, as well as the difficulty of developing mathematical work during a time period when mathematics underwent a considerable change from being based on 'time and space' to be considered in a purely conceptual way. The intention is also to discuss the importance of providing students the historical background to our modern mathematics.
Björling developed his mathematical work during a time period when mathematical analysis underwent a major change. Laugwitz (1999) describes this change as a 'turning point' in the ontology as well as the method of mathematics and argues that instead of using mathematics as a tool for computations, the emphasis fell on conceptual thinking. In my talk I will discuss examples where one can discern the 'old' mathematical approach as well as the 'new' mathematical approach in Björling's work. Björling had a tendency to sometimes consider mathematical definitions as descriptions of entities rather than conventions. For instance, he defined a function as [...] *an analytical expression which contains a real variable x* (Björling, 1852, p. 171). Björling certainly defined functions but sometimes he seems to consider the definition as describing something that already exists. As a consequence of his definition of a function, Björling considered every expression containing a variable that could be written up as a function. For instance, Björling argued that the function

$$f(x) = \frac{x}{|x|}$$

(expressed with modern terminology) attains the two values ±1 in x=0. But the derivative in the same point does not exist since the function representing the derivative jumps at x = 0. However, the derivative of the function g(x) = |x| at x = 0 exists and attains the two values ±1. A general problem for Björling was that his function defintion was too vague. In my talk I will show how jörling's vague function defintion led to ambiguities of how to consider complicated function expressions. Sometimes Björling (as well as his contemporaries) had to construct additional rules to the vague defintions in order to maintain a consistent theory. In a didactical perspective Björling's struggle with the definitions of fundamental concepts can give rise to fruitful discussions among students regarding the need for rigourous definitions in mathematics. When students first meet abstract mathematics problems often occur. The defintions can sometimes 3 seem too difficult and there is no "natural" explanation to lean on. Moreover, textbooks in mathematics rarely discuss the difference between mathematical and everyday objects, which can lead to the idea that mathematical objects always can be understood on the basis of "naturalistic" or physical arguments. Providing the historical background of the change that mathematics underwent during the mid 19th century can give students a better understanding of mathematics as a whole and that mathematics is constructed by people over a long time period.

Harald Gropp (Germany): ***Thomas Clausen (1801 - 1885) or how to become a professor of astronomy.***

This talk will discuss the Danish astronomer Thomas Clausen who was born in Snogbaek in Denmark in 1801 as a child in a poor farmer's family and died in Tartu in Estonia in 1885 as a professor emeritus of astronomy. He neither attended regular school nor did he study at a university.

However, he became the director of one of the important observatories of the Russian tsar. Clausen contributed to research in mathematics and theoretical astronomy although he was mainly trained as a practical astronomer and specialist in producing optical instruments with Fraunhofer in Munich and as an assistant of Schumacher in Altona.
This research is a sequel to my papers and talks on Clausen in the last years.
Particular aspects of this talk will be the problem of his birth and death dates, possibly still undetected results of Clausen, and altogether, the fate of an internationally active and interdisciplinary minded scientist in the historiography of mathematics and astronomy not much in the focus of interest in Denmark, Germany, and Estonia.

Room A130 (theme 6):

Karasawa Toshimitsu(Japan): ***Specialization and generalization of the mathematical concept by Mr. Inatsugi Seiichi in Japan.***

I describe the nature of the "life arithmetic" which was a mainstream in the arithmetic education of Japan in the 1930s. Mr. Shigeo Katagiri said about it that it was the ideal arithmetic education as an integrated part of the "life" and "mathematical."On the other hand, Mr. Tsutomu Okano said about it that it was to train a attitude of "arithmetic view and arithmetic ideal" by using a "typical life" material. However, both men are evaluating Mr. Inatsugi Seiichi. This point is very interesting for us. However, although both men cites the discussion of Mr. Inatsugi, Mr. Katagiri was positioned as the "true of life arithmetic." And, Mr. Okano was positioned as the highest reached point of the educational content research to oppose the black covered textbook which lead to the academic mathematics starting from the amount. Therefore, I want to clarify the flow from the arithmetic reform movement of children centricity to the formation process green covered textbook by considering the claim of Mr. Inatsugi in the 1920s and 1930s. As the result, the next thing became clear. Mr. Inatsugi insisted Inatsugi insisted is introduced the mathematical contents of geometry and algebra into the elementary school by the influence of mathematics teaching movement. On the other hand, he was opposed to the arithmetic education of the children living center that does not live up to the system of the subject. Mr. Inatsugi had the goal to connect mathematics to the arithmetic according to the logical system of arithmetic. He was not only to induct the arithmetic theory from the life experience of the children. He was able to justify the teaching contents of the mathematics and advanced arithmetic by including a further generalization of the theory which is generalized once. In this regard, Mr. Katagiri was regarded as integration "life" and "mathematical principle" about the claim of "specialization and generalization of mathematical ideas" by Mr. Inatsugi. By consideration of this, I found that the integration which was tried by Mr. Inatsugi was not a "living" and "mathematical principle." The integration he tried was the learning along the lines of the subject to teach the " children centricity" and "higher mathematics." Also, Mr. Okano overlooked this attempt by Mr. Inatsugi for he was not watching a claim of arithmetic education of the children centricity.I say about what was claimed "specialization and generalization of mathematical ideas", why at this time. In 1926, "Elementary School Ordinance Enforcement Regulations" was amended in Japan. The geometry and the algebra were introduced in the high school. The languages which are "an experiment and survey is used" and the " It is made to get used to handling of a chart, a table of compound interest, etc." was added in the elementary school. At the National Council of school teacher in1929, the intention of the new national textbook compiled to replace the black covered textbook had been shown from the Ministry of Education in Japan. The opinion of Mr. Inatsugi was an arithmetic educational theory aiming at integration a children centricity and the mathematics which thought the arithmetic theoretical system as important.

Hervé Renaud (France): ***When modern mathematics questioned the pedagogical methods in secondary education (1905-1910).***

In France in 1905 a controversy broke out between the mathematician Henri Poincaré and the philosopher and logician Louis Couturat. This controversy arose on the occasion of the publication by Couturat of a review of Bertrand Russell's book: *The principles of mathematics*. On the fringes of this debate Charles Lucas de Pesloüan published in 1907 four articles on the same topic. He was an engineer and a former student of the *École Polytechnique.* Those articles were published in the *Revue de Philosophie*, a journal which promoted the defense of the Christian philosophy.
Those four articles heavily criticized the algebraic logic supported by Couturat. Lucas de Pesloüan adopted some of Poincaré's arguments and added its own point of view. He broadened the discussion by criticizing a book of the French mathematician Hermann Laurent about the general principles of the number theory. In 1909 those articles were consolidated in a book. The preface of this book specified Lucas de Pesloüan's thought and also criticized a textbook of the French mathematician and pedagogue Jules Tannery.
From the outset Lucas de Pesloüan placed the debate on the mathematics education. If we combine those texts with some others (in particular the lectures given at the *Musée pédagogique* in 1904 after the important reform of the French secondary educational system in 1902) we can see that fears appeared of an introduction of modern mathematics in the secondary education. This study shows that the discussions about the mathematics principles affected the reflections on the secondary education of mathematics during this period.