Abstracts for poster presentations

Monday July 14

17:30-18:00: Poster presentations in five parallel sessions.

Room A303

Miglena Asenova and Giorgio Bolondi: *A hermeneutic approach to history and epistemology in mathematics education: the case of probability.*

In his work "Interpretazione e didattica della matematica: una prospettiva ermeneutica", G. T. Bagni (2009) suggests a shift from epistemology to hermeneutics in mathematics education. A hermeneutic approach offers the opportunity to "work with the positions incommensurate with the current discourse" (Bagni, 2009, p.16) and therefore allows to disengage from the need of coherence imposed by an epistemological approach to learning. As investigative tool, Bagni proposes an adaptation of Peirce's semiotics. At the basis of this semiotic we find the semiotic triangle but, from a global point of view, Peirce's semiosis is a potentially unlimited process that leads to the progressive construction of the meaning of a dynamic object. Bagni shows how the initial sign, which allows to start the semiotic chain, is comparable to an initial attitude (*habit*), and what Peirce calls "the final logical interpretant", can be seen as a mental "effect" (*habit change*) (Bagni, 2009, p. 212).
According to us, the hermeneutic approach proposed by Bagni is particularly interesting with regards to the role of history and epistemology in teaching not only mathematics but also in mathematics education (D'Amore, 2004; D'Amore, Fandiño Pinilla, 2013; Furinghetti, 2007) because it offers a useful tool in producing and interpreting changes of teachers' beliefs (D'Amore, Fandiño Pinilla, 2004; Goldsmith et al., 2014) related to mathematical knowledge, even when this requires a radical reference system reorganization.
Probability is a meaningful example in this sense, because its history has experienced at least two major epistemological fractures, which teachers not always seem to be aware of. The first, with Buffon (1777), in which bursts a situational substrate centered on continuous, as opposed to the classical discrete one, and which radically changes the operational tools necessary to investigate situations, replacing arithmetic by geometry. The second, with Kolmogorov, where the three approaches (classical, frequentist and subjectivist), which have irreconcilable philosophical and epistemological foundations, are exceeded by an axiomatic approach (Cera, 1990). So, we suppose that probability represents a very recent epistemological obstacle that arises from a cultural substract (the concept has a strong subjective e cultural based background) and could so contribute to an assessment of the debate between modern Theory of the obstacles in the classical sense coming from the interpretations given by Guy Brousseau (Perrin-Glorian, 1994) and Radford's Cultural Semiotics (D'Amore, Radford, Bagni, 2006). It's therefore we believe that the rejection many teachers have against probability, behind the superficial layer of insecurity due to initial training lacks (Paola, 1994; Stohl, 2005, pp. 345-366), may be ascribed to a week interpretation of the involved concepts. A reflection based on the study of historical documents, following a hermeneutic approach, centered on those that
historically have been the conflicts that preceded the axiomatization by Kolmogorov (Cera, 1990; Batanero et al., 2005), may allow to bridge the gap between different epistemological discourses

and could be an efficient intervention to produce a change in teachers' beliefs.

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Room A210

Davidson Paulo Azevedo Oliveira, Milton Rosa and Marger da Conceição Ventura Viana (Brazil):

The Contributions of Funds of Knowledge and Culturally Relevant Pedagogy as methodologies for the Development of Sociocultural Perspective of History of Mathematics in Mathematics Classrooms

This study is grounded in the Sociocultural Perspective of History of Mathematics, Funds of Knowledge (FoK), and Culturally Relevant Pedagogy (CRP) theories. It was conducted with the purpose of seeking contributions to activities based on gaining insight into parts of students' culture,

specifically, their FoK. The other purpose is to understand the role of History of Mathematics (HM) that can help teachers to comprehend students' questioning and reasoning about mathematics. The population was composed of 72 students from two classes in a first year of a technical course in a public technical high school in Ouro Preto in the state of Minas Gerais, Brazil. The researcher collected information that could answer the research question: *What are some of possible contributions that activities based on students' funds of knowledge and anchored in sociocultural perspective of History of Mathematics can bring to teaching and learning functions through the use of Culturally Relevant Pedagogy approach?* Two questionnaires, two focus groups, field notes, interviews and informal conversations with participants, and three documental records containing mathematics activities related to *functions* content were used. HM was applied in both implicit and explicit ways, which served as an orientation guide so that the researcher-teacher could develop the proposed activities by applying the FoK of participants, which helped in the analyses of the students way of represent and/or write functions concepts. We highlight the use of History of Mathematics in high school context in explicit and implicit ways. The implicit way let the teacher-researcher guide some activities and understand some of students' answers. On the other hand, the explicit way was used as problems taken from historyto be worked out by the students.It was found that the acquisition of mathematical knowledge and algebraic symbolic language in the classroom is related to students' cultural experiences. This approach allowed us to use some propositions of CRP, which is defined as a critical pedagogy that is committed to collectivity and is based on a tripod composed by critical awareness, cultural competence, and academic success. For data collection, analysis, and interpretation of qualitative and quantitative data, a mixed methods study *QUAN + QUAL* and content analysis were used. Data were collected and analyzed concurrently in all phases of thestudy. Thereafter, the results were analyzed, discussed, and interpreted in order to be addressed as part of the research. The interpretation of the results showed that the majority of participants learned and improved their knowledge in relation to symbolic algebraic notation by highlighting the importance of rhetoric stage of algebra in order to understandsymbolism and academic development of symbolic algebraic language. Besides that, we drew attention to the fact that History of Mathematics used in explicit way cannot be applied as a teaching methodology for high school teachers in all mathematical content. However, it can be used in an implicit way to help teachers to understand students' reasoning even though the sociocultural context is very important in this understanding.

Julio Corrêa (Brazil/Denmark): *Mathematics, education and war*

In this work we present some ideas related to a PhD project started in 2011 where we try to problematize some relations between mathematics, education and war. More precisely we have been trying to understand the modifications in the field of mathematics education in the context of Cold War. Here we try to problematize the enigmatic phrase of Jean Dieudonné: "Euclid must go!". For a long period in the history of mankind the Euclidian geometry played an important role not only in the warfare, but at the schools and in science in general. So, why, in a context where the "Western" headed by United States seemed to be losing the conflict capitalism versus communism, Euclid must go? Tracing some relations between the development of disciplines as Operational Research, Game Theory, Linear Programing, the emergence of computer sciences and the structuralist mathematics proposed by the Bourbaki group, we shall enlighten the so called "modern mathematics movement" and its relation with the Cold War. Apparently there is no explanation available for the demise of Euclidian geometry and the predominance of "New Math" solely in terms of mathematics neither in terms of society neither of mathematics education, then

we need to look for fields of human activities and its relations to explain this event. Based on a post-structuralist theoretical approach, mainly on the works of the "second" Wittgenstein, Jacques Derrida and Michel Foucault, we try to develop a *grammatical deconstructive therapy* of mathematics, education and war in the specified context. We believe that such kind of historic-philosophical problematization could help teachers to understand the relations between the field of mathematics and other fields of human activity which may help them to show to students the role of mathematics in different contexts of socio-cultural practices.

Room A212

Michela Maschietto (Italy): *From history to primary classrooms with B. Pascal: approaching place value and arithmetical operations with pascaline and e-pascaline*

Le poster concerne des expérimentations didactiques centrées sur l'utilisation d'artefacts matériels et virtuels dans la construction de significations mathématiques à l'école primaire (Bartolini Bussi & Mariotti 2008), suivant la méthodologie du laboratoire de mathématiques (Maschietto & Martignone 2008). En particulier, ce poster présente l'utilisation d'une machine arithmétique (Zero+1), construite en plastique à fonctionnement mécanique, et sa version numérique. Ces machines évoquent la fameuse machine Pascaline réalisée par Blaise Pascal. Pour cette raison, elles ont été nommées respectivement pascaline et e-pascaline.
La machine matérielle (Maschietto 2013) a été utilisée dans des classes d'école élémentaire et du début de l'enseignement secondaire pour travailler: l'approche récursive aux nombres naturels, l'écriture en notation positionnelle décimale, les opérations arithmétiques et quelques propriétés des nombres naturels. L'introduction de la pascaline dans les classes permet aussi de traiter la figure de Blaise Pascal and l'histoire des machines à calculer. Par exemple, des sélections de la lettre de présentation de la machine écrite par Pascal ont être proposées à la lecture collective des élèves.
À partir de l'objet matérielle, nous avons réalisé sa contrepartie numérique dans le cadre du Projet français Mallette de l'IFé-ENS de Lyon (http://educmath.ens-lyon.fr/Educmath/recherche/equipes-associees/mallette/mallette-lyon/mallette). Avec la e-pascaline, nous avons conçu des cahiers informatiques dans l'environnement Cabri Elem (Maschietto & Soury-Lavergne, online first). Dans notre perspective, la pascaline et la e-pascaline sont étudiées en termes of duo d'artefacts, nous permettant d'une part de questionner les relations entre les deux types d'artefacts du point de vue de l'apprentissage des mathématiques, d'autre part de répondre à des demandes institutionnels de ressources pour l'enseignement des mathématiques.
Les données des premières expérimentations avec le duo d'artefacts seront inclus dans le poster. Des exemplaires de la pascaline accompagneront le poster.

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Room A214

Leticia del Rocío Pardo-Mota and Alejandro Rosas-Mendoza (Mexico): *Math for Special Education.*

In this poster we address a brief chronology of schools and colleges in Mexico that included mathematics courses aimed at the student population with disabilities.

Towards 1809, 1821 and 1830 were opened some private educational programs for deaf education. Eduardo Adolfo Huett Merlo on February 14, 1867 founded the Municipal School of Deaf in Mexico City and classes began in March 1867 with 12 students, but at the beginning only studied grammar and writing. On November 28, 1867, President Benito Juarez transformed it in the National School of Deaf. It had two areas, the first one is an elementary school for the deaf (they learn arithmetic) and a normal school for teachers who teach deaf.
On March 24, 1870 Ignacio Trigueros y Antigua achieved the creation of the National School for the Blind. Between December 1st, 1890 and March 3rd, 1891, the National Congress for Public Education concluded that it is necessary to increase the number of special schools that were aimed at education of the blind, deaf and young offenders. This point became more relevant after the Law of Basic Education on August 15th, 1908.
In 1915 was founded in the city of Guanajuato the first school to serve children with mental deficiency, subsequently it diversified its services to children and youth with different disabilities.
In January 1925 the government founded the Department of Psychology and School Hygiene in order to investigate and learn the mental development of Mexican children who had school delay (learning problems). This institution starts the application of tests to measure intelligence and determine who can learn and who cannot. It created a classification for "abnormal children" with six divisions: children with language disorders, children "hard of hearing", visually impaired children, locomotor disabled children, children with epilepsy and children with tuberculosis.
Towards 1940 officials began with the idea that schools for "physically or mentally abnormal children" should provide in short cycles, a curricula including general knowledge, programs, teaching methods and particular organization.
The poster will include pictures and related information from those periods of time.

Leticia del Rocío Pardo-Mota and Alejandro Rosas-Mendoza (Mexico): *Progressions and series in ancient China*

In this poster we present a brief chronology of the series numerical progressions and Chinese mathematics.

The arithmetic and geometric progressions appear in China in a book called 九章算術 or Jiuzhang suanshu (Chu Chang Suan Shu) or Nine Chapters on the Mathematical Art , written approximately around 200 BC (Some historians assigned a date between 100 BC and 50 a. C.), and which over time mathematicians were adding various comments. In Chapter 3, named Cui fen or Distribution by Proportion, there are problems whose solution involves the use of arithmetic and geometric progressions.
Zhang Qiujian (also known as Chang Ch'iu-Chin or Chang Ch'iu -chien) wrote a book called Zhang Qiujian suanjing (Zhang Qiujian's Mathematical Manual) between 468 d. C. and 486 d. C.

(Some historians date Zhang Qiujian 100 years before) consisting of 98 problems divided into three chapters. In this work solves and computes the sum of arithmetic progressions
Zhu Shijie also known as Chu Shih-Chieh was born around 1260 near Beijing, China. It is known that he wrote two works considered as surprising. The first named Suan xue qi meng (Introduction to mathematical studies) published in 1299 and came to be used as a textbook of mathematics in Japan (printed in 1658) and Korea (printed in 1660) deals with polynomial equations and polynomial algebra , areas, volumes , rule of three and a method equivalent to Gaussian elimination . The second book published in 1303 is Siyuan yujian (True Reflections of the Four Unknowns) and it includes the famous Pascal's Triangle to the eighth power. He solves polynomials with 1, 2, 3 and 4 unknowns. It also features 288 problems divided into three volumes with 24 chapters. He presented formulas of sums of integers like

$1+4+10+20+\ldots+\frac{n(n+1)(n+2)}{6}=\frac{n(n+1)(n+2)(n+3)}{24}$ among others, and also provided the sum of series of the kind $1+4+9+16+25+36+\ldots$, $1+5+14+30+55+91+\ldots$
Yang Hui was born in 1238 in Qiantang (present Hangzhou, China). In 1261 Yang wrote the Xiangjie jiuzhang suanfa (Detailed analysis of the mathematical rules in the Nine Chapters and Their reclassifications). Yang also gave the Pascal Triangle's scheme to the sixth row and also gave formulas for the sum of series like $1+3+6+10+\ldots+\frac{n(n+1)}{2}=\frac{n(n+1)(n+2)}{6}$ and the sum of the squares of the natural numbers between m2 and (m + n)2.

Room A104

Zhu Liu (China): *Genetic approach to teaching derivative*

Calculus is the main mathematical subject taught in both senior high school and university. However, the teaching of the concept of calculus is universally difficult. A historical and epistemological analysis of calculus is a way to reveal some possible sources of students' difficulties as well as an inspiration in the design of activities for students. Otto Toeplitz first summarized and elucidated calculus of the history of mathematics as an organic evolution of ideas beginning with the discoveries of Greek scholars and developing through the centuries in his book< The Calculus: A Genetic Approach>. The genetic approach to teaching and learning is that a subject is studied only after one has been motivated enough to do so, and learned only at the right time in one's mental development.
The problem of the tangent line is one of the most important problems which lead to the birth of calculus. Through an questionnaire survey conducted to 332 students, we concluded that there are historical parallelism between the students' understanding and that of the ancient Greek mathematicians. On the base of historical and epistemological analysis of the concepts of derivative, we design a teaching instruction by integrating history of the birth of calculus, such as problems of light reflection, curve movement. Based on the reconstructed history, The Cyclotomic Rule by Liuhui is introduced to construct a bridge connecting the static and dynamic concept of the tangent, enabling students to pass from the finity to infinity naturally and successfully. It is revealed through interview and a questionnaire survey that the genetic approach to teaching derivative in conducible to better understanding of the concepts of derivative.

Laurence Kirby (USA): *Plimpton 322: A video documentary to motivate students to study mathematics*

I shall provide an introduction to the 33-minute film *Plimpton 322: The Ancient Roots of Modern Mathematics*, which was produced to motivate college and high school students – especially minority students – to pursue mathematics.

The mathematics that drives our modern world owes its origins to ancient cultures in the Middle East, Asia and Africa. Set against a backdrop of today's New York City, the film explores the extent of our debt to this tradition. Along the way we meet up close some precious and revealing ancient artefacts that now have their homes in New York, most of all a controversial cuneiform tablet from Mesopotamia known as Plimpton 322. We witness ancient mathematical ideas still playing crucial roles in 21st-century society and technology.

This film celebrates the diversity underlying our mathematical culture. Teaching at a large, urban, multiethnic university, I have found it useful in encouraging students in courses ranging from mathematics for liberal arts students to history of mathematics.

The film incorporates brief introductions to two mathematical topics, positional number notation and Pythagorean triples, which can be developed in the classroom.

The purpose of my presentation is to bring this freely available resource to the attention of educators and to discuss with anyone interested its use in the classroom. As well as a poster, I shall display a trailer and excerpt from the film on a laptop computer or tablet.

The film is at http://faculty.baruch.cuny.edu/lkirby .