Can a "Price-Distorting" Tax Help Meeting Climate Change Targets by Speeding Up Substitution in Natural Resources Use?

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RESOURCE OWNERS AND RESOURCE USERS. DIFFERENTIAL EFFECTS OF TAXING THE USE OF POLLUTING EXHAUSTIBLE RESOURCES

- **Resource owners** - Consider a (representative) resource owner of a given stock S_0 of an exhaustible (polluting) resource. The resource is sold in a competitive market with (inverted) demand $p(x) = x^{-\theta}$, and is extracted at the rate $x(t)$ with a linear extraction cost cx. Our optimal control format is not one of some type of social welfare optimization. We simply assume that the resource owner maximizes the present value of the infinite stream of his competitive profits, under the constraint of the given stock S_0, and then investigate how a tax on the profits of the resource owner affects the optimal time path of the control variable, the extraction rate $x(t)$.

- **Resource users** - Consider a (representative) resource user who uses the resource as an input for producing an output according to a production technology $q(x) = x^{\alpha}$ and the same linear extraction cost function cx. As before, we assume that the resource user maximizes the present value of the infinite stream of his competitive profits, under the constraint of the given stock S_0, and then investigate how a tax on the profits of the resource user affects the optimal time path of his profits, via its effects the control variable $x(t)$.

We summarize by points the main facts arising out of the working of the model, as they appear to us at this preliminary work-in-progress stage of the analysis. We are currently working to develop, generalize and refine the mathematical properties of the model, in order to produce more rigorous proofs of the described facts, and more precise conditions under which they hold.

1. Resource Owners

$$max \int_{t=0}^{\infty} [p(x(t)) - c] x(t) e^{-rt} dt$$

s.t.

$\dot{S} = -x(t), \quad S(0) = S_0, \quad S(t) \geq 0, \ \int_{t=0}^{\infty} [x(t)] dt = S_0$

where $p(x) = x^{-\theta}$

- Current Value Hamiltonian:

$$G_x(\cdot) = H(\cdot)e^{-rt} = [p(x(t)) - c]x(t) + \lambda(t)(-x(t))$$

- Optimality conditions:

$$\frac{\partial G_x}{\partial x} = 0 : \ [p(x(t)) - c] = \lambda(t)$$

$$\dot{S} = \frac{\partial G_x}{\partial \lambda} \ : \ \dot{S} = -x(t)$$

$$\dot{\lambda} = -\frac{\partial G_x}{\partial S} + r\lambda(t) : \ \dot{\lambda} = r\lambda(t) \ \rightarrow \lambda(t) = \lambda_0 e^{rt}$$

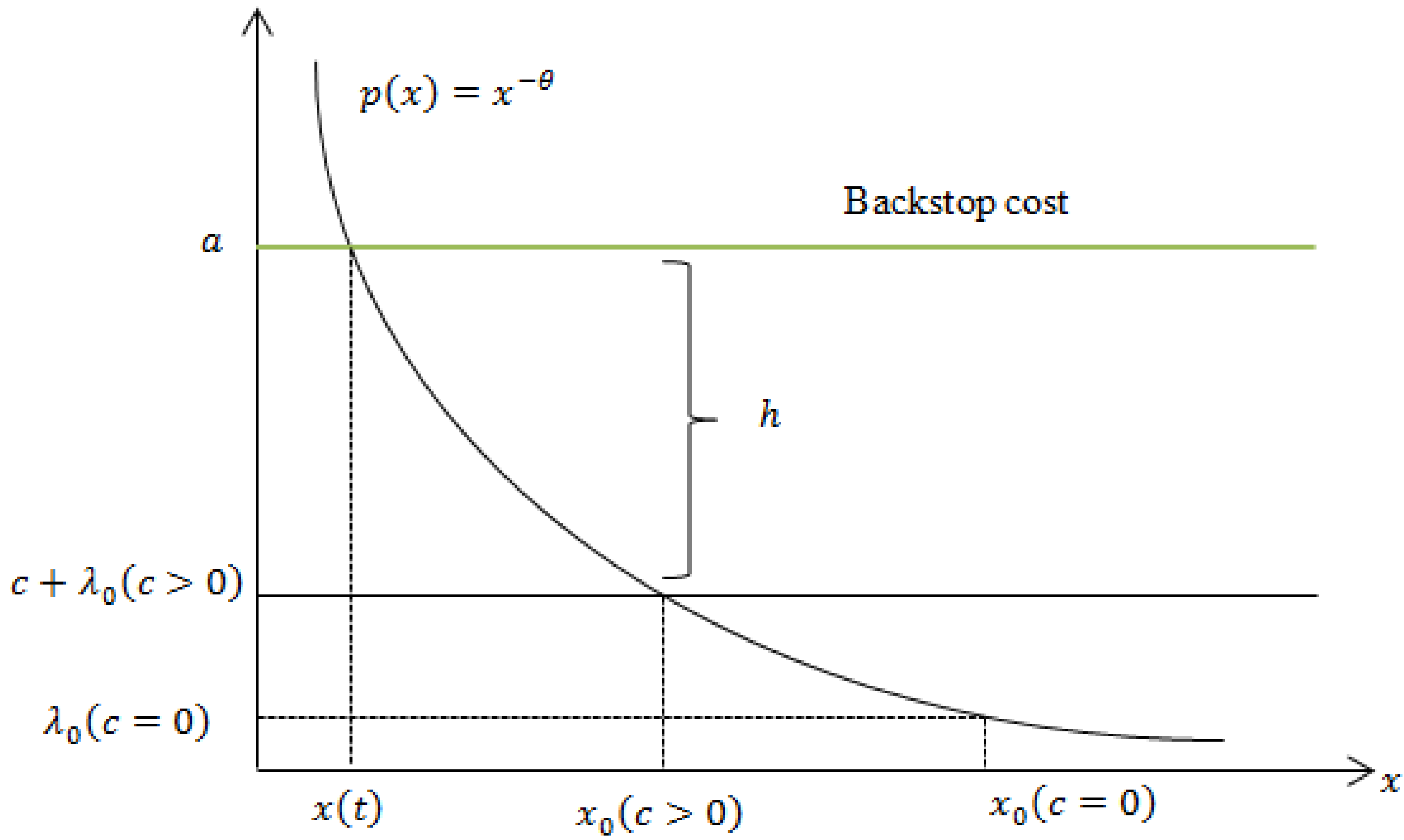

Figure 1: Non-renewable extraction rate

For the sole purpose of using it as a benchmark, we consider the situation where extraction costs are zero: $c = 0$. The nice property of this benchmark case is that yields a well-defined analytical solution for the optimal time path of the control variable $x(t)$ (equation (1) and figure 2):

The optimal time path of the proportionate extraction rate:

- if c=0 $\rightarrow \frac{\dot{x}}{x} = -\frac{r}{\theta}$,

then $\int_{t=0}^{\infty}[x_0]e^{-\left(\frac{r}{\theta}\right)t}dt = S_0 \rightarrow \frac{x_0}{r/\theta} = S_0 \rightarrow x_0 = S_0 \cdot \frac{r}{\theta}$

- while with a positive c: **(1)** $\frac{\dot{x}}{x} = -\frac{r}{\theta} \cdot \frac{p(x)-c}{p(x)}$,

The simplest way to introduce a tax on the resource owner's profits is to represent it as a unit tax h to be added to c and yielding the increased (linear) cost function $(c + h)x$. We therefore do not actually need to formally specify the tax, because it is enough to simply look at how an increase in c , from zero upwards, affects the optimal (for the resource owner) time path of the control variable $x(t)$.

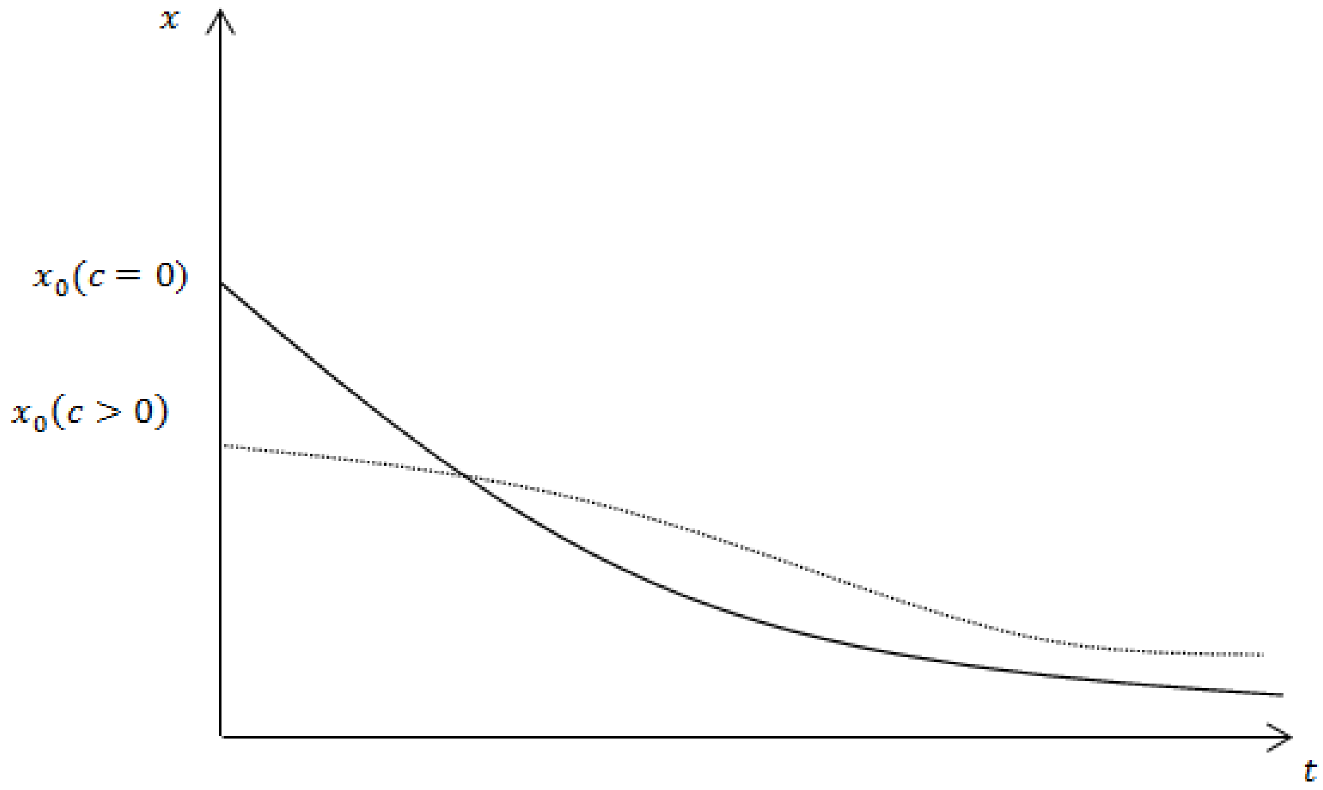

Figure 2: Time path of extraction

Some emerging facts

1) With $c = 0$ we obtain the above solution for the optimal. As c increases we can see that the optimal path $x(t)$ changes in a qualitatively well defined manner. First, its initial rate of decrease decreases, becoming lower (in absolute value) than $-(r/\theta)$, and then increases (in absolute value) over time, approaching $-(r/\theta)$ as $t \to \infty$. This is shown by **eq. (1)** and in **Figure 2**. In reading **Figure 2** we must keep in mind the constraint of resource stock exhaustion at $t \to \infty$ given in **eq (1)**.

Thus, with a positive c the optimal path $x(t)$ shifts downward initially, but then, at some time t, crosses above the benchmark path (remember that the value of the integral $\int_{t=0}^{\infty}[x(t)]dt$ must remain unchanged because it represents the resource stock exhaustion). In short, what happens when c increases (remember that we are interpreting this as caused by the introduction of a unit tax h) is that the rate of extraction becomes – so to speak – more stable. It keeps decreasing over time, but at a slower pace.

In passing, as a pure mathematical point, we mention that within the given model it should be possible to show that as $c \to \infty$ the extraction rate tends to become constant, which implies, in the limit, a finite terminal time of full resource depletion.

follow

2) Clearly, the assumption that the current price $p(x)$ of the resource has no upper bound and $\rightarrow \infty$ as $x \rightarrow 0$ makes no sense. As a first approximation, we assume the existence of some backstop technology which sets an upper bound to $p(x)$, shown in **Figure 1**. If so, what the model shows is that, given some $c + \lambda(0)$, the decrease of $x(t)$over time, and the associated increase of $(x(t))$, encounter a limit, represented precisely by that upper bound. When $p(x(t))$ reaches that upper bound (or, to be precise, just one ε below it), $x(t)$ ceases to decrease. It remains constant over time at that particular level, with the consequence that the resource stock S_0 will be exhausted within a finite terminal time. In other words, the assumption of some upper bound to $p(x)$ (a backstop technology) means simply that sooner or later $x(t)$ ceases to decrease, and the resource will actually be fully depleted in finite time. The raising of c through taxation, by 'stabilizing' (relatively speaking) the $x(t)$ path, may shift to a later date the beginning of the constant rate of extraction, but the resource will in any case be fully depleted in finite time.

3) Under our assumptions of a constant given tax (possibly also *ad valorem*, though we haven't formalized this case), the model shows that if we want the use (extraction) of the polluting resource by the owners to stop, then we must introduce a tax that raises $c + \lambda(0)$ to the point where it exceeds the upper bound level of $p(x(t))$. In other words, the tax must be sufficiently high as to make the extraction unprofitable. As long as it remains profitable, the existence of a backstop technology simply means that the extraction rate will deplete the resource in finite time.

2. Resource Users

$$max \int_{t=0}^{\infty} [q(x(t)) - cx(t)]e^{-rt} dt$$

s.t.

$\dot{S} = -x(t), \quad S(0) = S_0, \quad S(t) \geq 0, \quad \int_{t=0}^{\infty}[x(t)]dt = S_0$

where $q(x) = x^{\alpha}$

- Current Value Hamiltonian:

$$G_x(\cdot) = H(\cdot)e^{-rt} = [q(x(t)) - cx(t)] - \mu(t)(x(t))$$

- Optimality conditions:

$$\frac{\partial G_x}{\partial x} = 0 : \ [q_x'(x(t)) - c] = \mu(t)$$

$$\dot{S} = \frac{\partial G_x}{\partial \mu} \ : \ \dot{S} = -x(t)$$

$$\dot{\mu} = -\frac{\partial G_x}{\partial S} + r\mu(t) : \ \dot{\mu} = r\mu(t) \ \rightarrow \mu(t) = \mu_0 e^{rt}$$

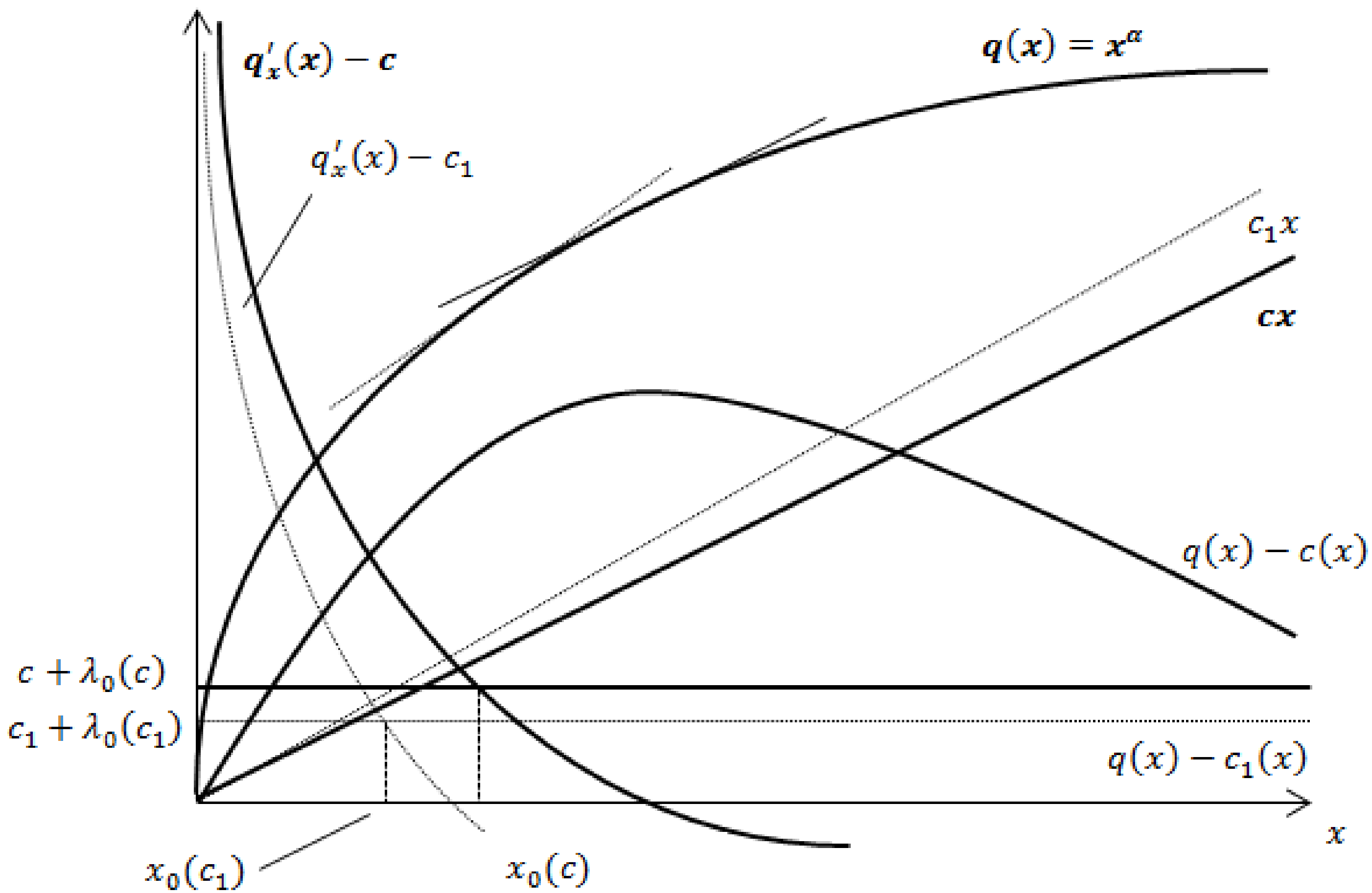

Figure 3: Resource users

To shorten the presentation we skip the repetition of the benchmark case, which is obtained in a way similar to that used in the previous model, with the difference that there we start directly with a constant elasticity demand function, while here we should start with a constant elasticity production function $q(x) = x^{\propto}$
The tax on the resource user's profits is introduced in the same way of the previous model. An increase in the tax is represented simply as an increase in c.

The facts emerging from this optimal control model are the following.

1) With we obtain a given optimal path $x(t)$ with an initial value $x_0(c = 0)$ and decreasing over time so as to exhaust the resource stock S_0 for $t \to \infty$. As c increases we can see that the optimal path changes in the same qualitatively well defined manner of the previous case. The initial value decreases and the rate of decrease becomes slower, so that the path becomes – so to speak – more 'stable', and at some point in time crosses above the benchmark path. This is shown in **Figure 4**. In the limit, for $c \to \infty$, the path tends to become constant and the exhaustion terminal time finite.

2) But what we are interested here is not the $x(t)$ path itself, but the path of the associated current profit $q\big(x(t)\big) - cx(t)$. This path is of course strictly associated to the extraction path. Starting with a profit path for some given cost c we can see that if c increases (because of taxation) the profit path shifts downwards, crossing the previous path at some point of time far to the right of the crossing point of the extraction paths. Of course, while the value of the integral of the changing extraction path remains unchanged, the value of the integral of the profit path decreases.

3) Now suppose there is another resource – say V_0 – that can be used for producing the same output (with extraction $y(t)$), and which is cleaner. We may represent the extraction path and the profit path for this new resource. In general, for some given extraction cost, we may have four simple cases: (i) the profit path of the cleaner resource lies entirely above that of the dirtier, (ii) it may lie entirely below, (iii) it may cross from below, (iv) it may cross from above. The user's choice will be the one that maximizes the present value of his profit, which, in the case of crossing, will mean switching from one to the other at the crossing time. Since an increase in the extraction cost (through taxation) shifts downwards the profit path, we see in Figure 4 that such tax-induced cost increase may cause to different effects. In case (iii) (the clean path crosses the dirty path from below) taxing the dirty resource shifts the crossing point to an earlier time: the producer moves earlier to the cleaner resource – as desired. In case (iv) taxing the dirty resource shifts the crossing point to a later date: the producer moves to the dirtier resource at a date which becomes later and later as the extraction cost keeps increasing.

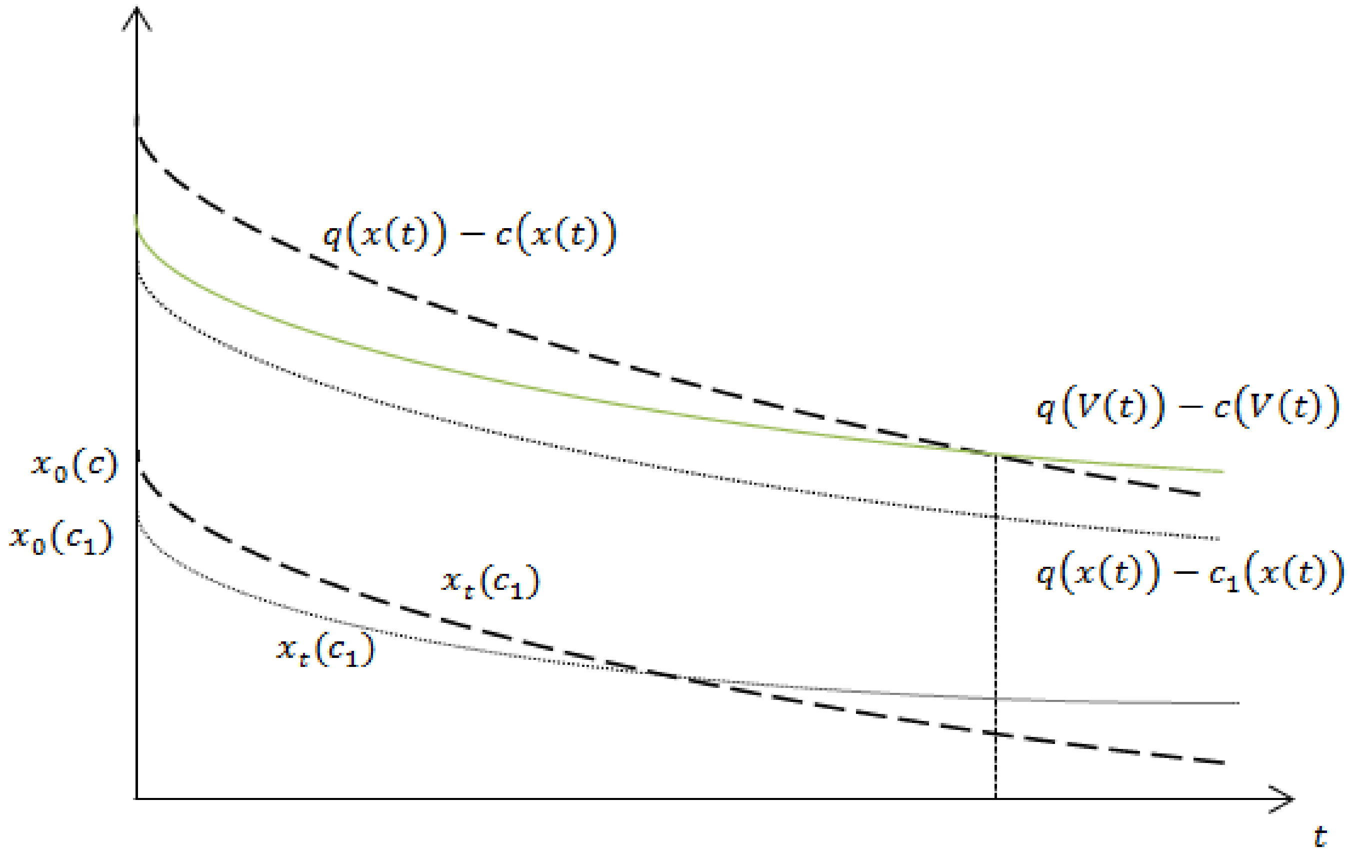

Figure 4: Extraction and profits paths

Conclusions so far

- Owners scenario:
- 1. with a unit tax initial extraction "decreases"
- 2. with a unit tax the extraction path follows a lower pace (i.e. more stable)
- 3. once a backstop is introduced extraction path becomes constant when the price reaches the upper bound (the stock is depleted in finite time)
- 4. with the tax (equal to the difference between backstop and current cost) extraction stops at time 0
- Users scenario
- 5. ……..tax makes the user to switch earlier to the cleaner input
- 6. ……….or it makes the user to switch later to the dirtier