Multipole radiation of oriented and aligned atoms

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Unlike unpolarized and isotropic radiation of nonpolarized atoms, the radiation of spin-polarized (oriented and aligned) atoms becomes anisotropic and, in general case, partially polarized that opens new possibilities for spectroscopic studies. In the present paper we derive the compact formulas that allow studying both the angular distributions and polarization peculiarities for the radiation of arbitrary multipolarity 2^l $(l = 1, 2, ...)$ in case of axially symmetric spin-polarization of a radiating atom. Such polarization arises as a result of unequal population of the atomic states with different projections of angular momentum on some direction ***n*** and is completely specified by 0-components ρ_K^n of *K*-rank state multipoles [1] ($K = 1, 2, ..., 2J_i$, where J_i is the angular momentum quantum number). The dependence on geometric parameters – the unit vector ***n***, determining the symmetry axis of the polarized atom, and ***k***, specifying the direction of radiation, as well as the unit complex vector ***e*** specifying the photon polarization – is completely separated out in the derived formulas.

The state multipoles of only even rank *K* = 2, 4,…, 2*l* (i.e. the alignments of different orders) contribute to the angular distribution of the radiation, the angular distributions of electric (*El*) and magnetic (*Ml*) 2^l-pole radiations coinciding. Thus, for the probability per unit time of electric quadrupole (*E*2) radiation in the direction of ***k*** we derive the following expression

$$\frac{\omega^5}{12\pi\hbar c^5}\left|\left\langle J_i \| Q_2 \| J_f \right\rangle\right|^2 \left[\frac{1}{5(2J_i+1)} - \frac{(-1)^{J_i+J_f}}{\sqrt{14}}\left(\rho_2^n P_2(x)\begin{Bmatrix} 2 & 2 & 2 \\ J_i & J_i & J_f \end{Bmatrix} + \frac{4}{\sqrt{5}}\rho_4^n P_4(x)\begin{Bmatrix} 2 & 2 & 4 \\ J_i & J_i & J_f \end{Bmatrix}\right)\right].$$

Here ω is the frequency of the radiation, J_i and J_f are the atomic angular momentum quantum numbers in the initial and final states, $\left\langle J_i \| Q_2 \| J_f \right\rangle$ is the reduced matrix element of the electric quadrupole moment operator, $P_n(x)$ is Legendre polynomial and $x = \mathbf{n}\cdot\mathbf{k}$. The expression for the probability of *M*2-radiation can be derived by substitution of magnetic quadrupole moment operator for electric one.

The Stokes parameters determining the polarization of *El* and *Ml* -radiations are interrelated: the degree of circular polarization $\eta_2^{Ml} = \eta_2^{El}$ and the degrees of linear polarization $\eta_{1,3}^{Ml} = -\eta_{1,3}^{El}$. The degree of circular polarization is only nonzero under orientation of the radiating atom, i.e. $\rho_K^n \neq 0$ for odd *K*, and the degrees of linear polarization are determined by alignment of the atom. Interesting polarization peculiarities of the radiation have to arise owing to interference of *Ml* and *E*(*l*+1)-amplitudes that can only be observed under spin-polarization of the radiating atom. In such a case the pseudoscalar $\mathrm{Re}\left[\mathbf{e}\cdot(\mathbf{n}\times\mathbf{k})(\mathbf{e}^*\cdot\mathbf{n})\right]$ enters the expression for probability of radiation and the degree of linear polarization becomes dependent on the higher-order orientations $\rho_3^n, \rho_5^n, ..., \rho_{2l+1}^n$ of the spin-polarized atom.

References:
[1] K. Blum, *Density Matrix Theory and Applications* (Plenum, New York, 1996).